Introduction to Probabilistic Methods with Applications to Probabilistic Damage Tolerance Analysis

PDTA Case Study

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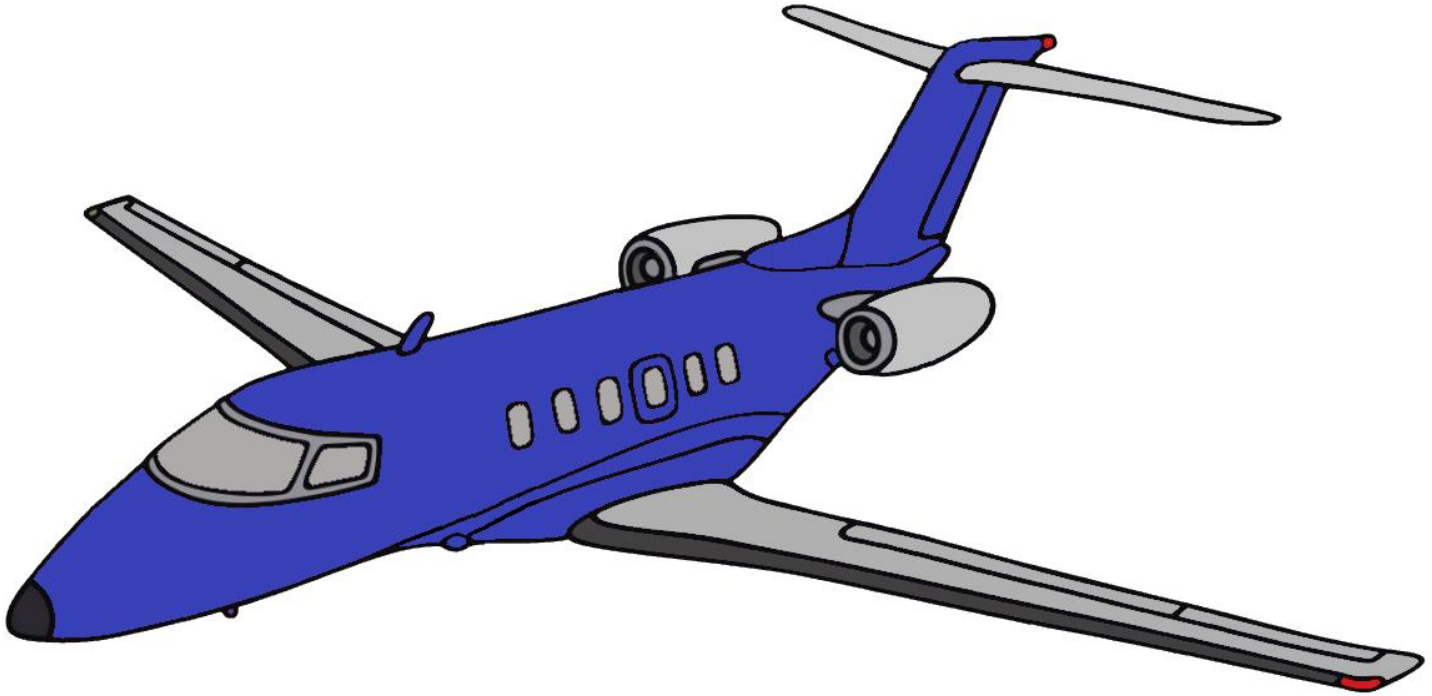
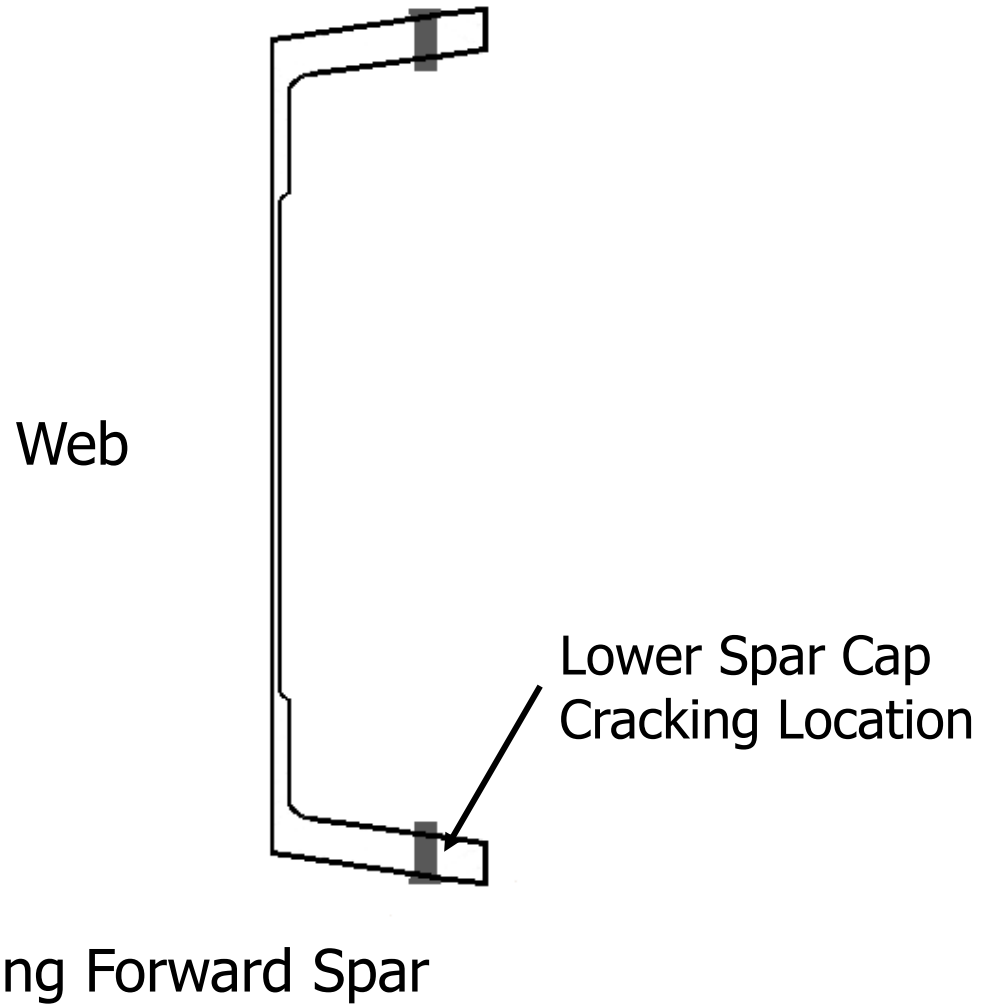
Problem Overview



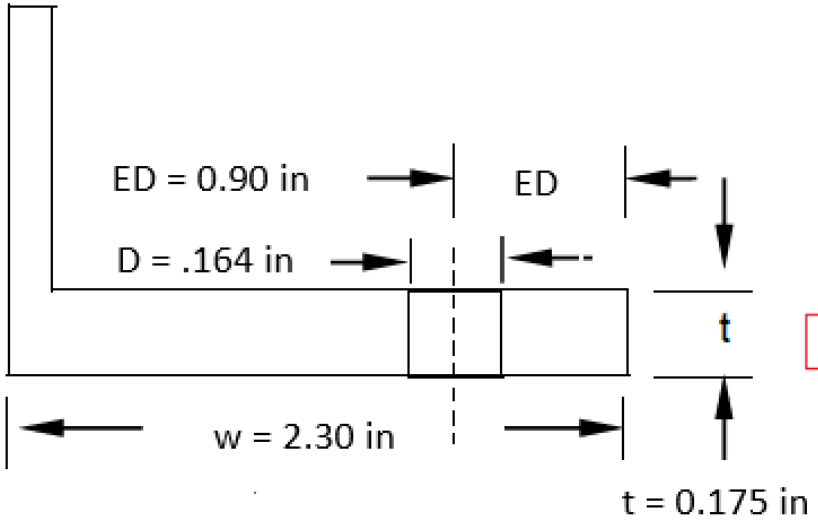
General Aviation Corporate Jet With
Wing Forward Spar Cap Cracking

Problem Overview

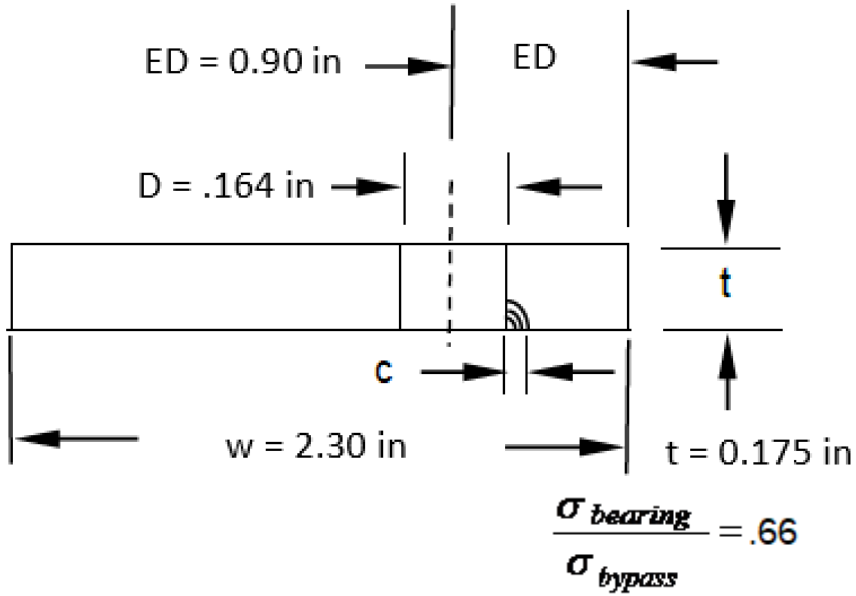


Problem Overview

Simplified Geometry



Idealized Geometry



$$\frac{\sigma_{bearing}}{\sigma_{bypass}} = .66$$

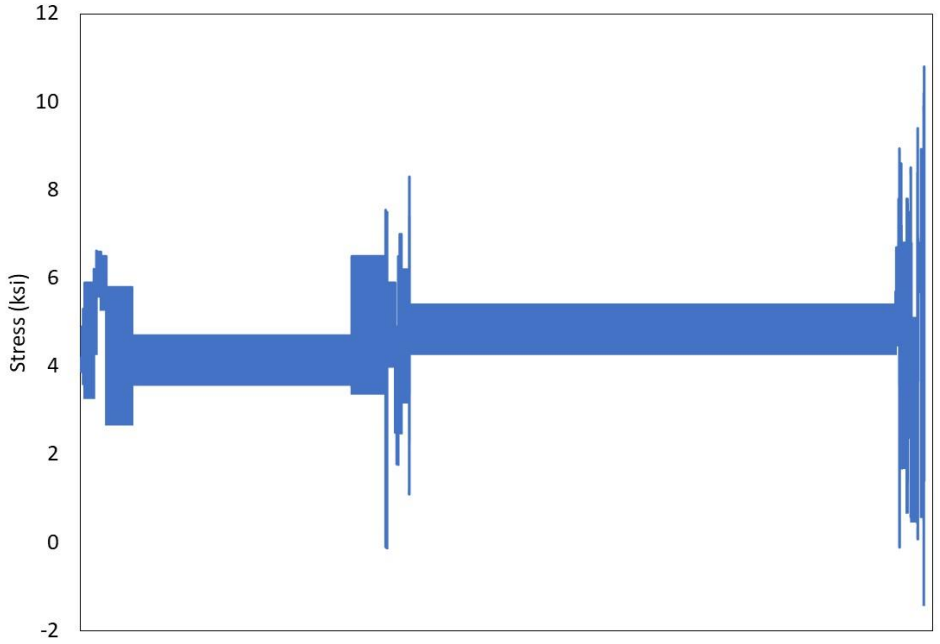
CLASSICAL CRACK GROWTH

Classical Crack Growth

- Determine initial and recurring inspections using classical crack growth analysis from .05” flaw
  - AFGROW software used for crack growth
    - User defined spectrum
  - Detectable crack length = .20 in.
  - Inspections
    - Initial inspection = $\frac{\text{Life}}{2}$
    - Repeat inspection = $\frac{\text{Life} - \text{Life@detectable}}{2}$

User Stress Spectrum

- User defined spectra
  - Gust cycles are based on PSD continuous gust criteria in 14 CFR 25 Appendix G
  - Maneuver cycles are based on NASA measured data for business jets
  - Spectra is in AFGROW format (1,000 flight hours, 1 flight = 1.65 Flt. Hrs.)
  - Used SMART to create a simplified GAG spectrum with equivalent damage



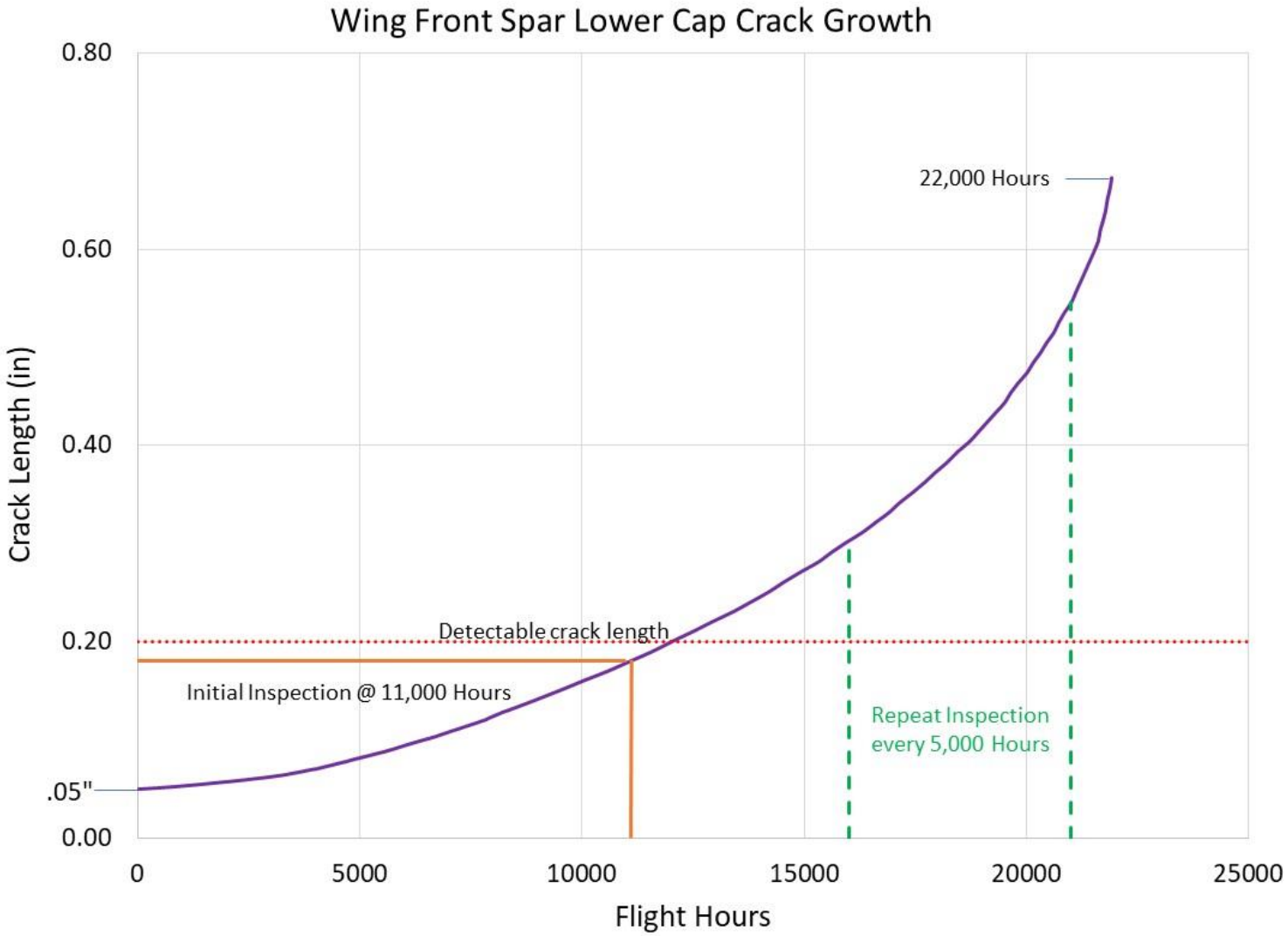
50 different max-min stress pairs, 33,600 total pairs

Classical Crack Growth

Variable	Parameter
Initial Crack Size	0.05 in
Fracture Toughness	37.0 ksi√in
Paris m	2.586
Paris c	1.29E-8
Walker exponent	.82
Ultimate Stress	69.0 ksi
Yield Stress	58.0 ksi
Hole Offset	0.90 in

AFGROW INPUT
Mean only

Classical Crack Growth



MASTER CURVE WITH AFGROW

Master Curve

- Master curve used for probabilistic analysis
  - Requires crack growth curve from crack growth software using mean spectra
    - AFGROW used to generate master crack growth curve
  - Probabilistic variables are initial crack size and fracture toughness
  - User inputs crack growth curve & residual strength information as userfile.avsn file to SMART

- Example file provided: Capstone_mastercurve.smdt

Analysis Location Properties

Random Variable	Distribution	Parameters
Initial Crack Size	Lognormal	Mean = 0.009055 in Standard Deviation =0.001252 in
Fracture Toughness	Normal	Mean = 37.0 ksi√in Standard Deviation = 3.8 ksi√in
Paris m	Binormal	Mean = 2.586 Standard Deviation = 0.0
Paris c (log)	Binormal	Mean = -7.888 Standard Deviation = 0.0
Coefficient of Variance		0.0
Walker exponent		.82
Ultimate Stress	Normal	Mean = 69.0 ksi Standard Deviation = 0.0 ksi
Yield Stress	Normal	Mean = 58.0 ksi Standard Deviation = 0.0 ksi
Hole Offset	Normal	Mean = 0.9000 in Standard Deviation = 0.0 in

SMART INPUT
Mean & StdDev

AFGROW INPUT
Mean only

SMART|DT – Project Information

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Information

Website

Information

Provide information about the project.

Project Summary

NAME (REQUIRED)

Capstone

DESCRIPTION

Front Spar Analysis

Aircraft Information

MAKE (OPTIONAL)

Acme

MODEL (OPTIONAL)

Sky Runner

SERIAL NUMBER (OPTIONAL)

All

TYPE CERTIFICATE DATA SHEET - TCDS (OPTIONAL)

TCDSSR1

This program was developed under sponsorship from the Federal Aviation Administration (grants 12-G-012 and 16-G-005) by the University of Texas at San Antonio (UTSA) and partners St. Mary's University, Textron Aviation, Nuss Sustainment Solutions, and Fieldstone Software. The responsible personnel are: Harry Millwater (PI - UTSA), Juan Ocampo (StMU), Beth Gamble (TA), Chris Hurst (TA), Marv Nuss (NSS), JR Lawhorne (Fieldstone), Nathan Crosby (UTSA PhD student), Daniel Ocampo (UTSA MS student), Sohrob Mattighi (Program Manager FAA), Mike Reyer (FAA Kansas City Office).

SMART|DT – Analysis Information (Output Options)

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Analysis

- Output Options
- Growth
- Probabilistic

Probability of Failure (POF)

Flights	Maximum Flights Calculation	Flight Units
50	30000	Flights

SMART|DT – Analysis Information (Growth)

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Analysis
- Output Options
- Growth
- Probabilistic

Model: Master Curve **Source:** User Generated

Master Curve

AVSN FILE
capstone1.avsn [Browse] [View Text] [P

MASTER CURVE FRACTURE TOUGHNESS
37.0

MASTER CURVE YIELD STRENGTH
64.0

HOURS PER FLIGHT
1.65

FAILURE CRITERIA
KC

SMartJDT - Avsn Content

```
Fracture_Toughness = 37.0
Yield_Strength = 64.0
Hours_Per_Flight = 1.65
Failure_Criteria = KC

flt_no  a              c              a1  c1  RS_by_Kc          RS_by_NSY
0       0.003          3.0000E-03     x   x   1.6363E+02        1.6363E+02
100     0.003000269    3.0001E-03     x   x   1.6362E+02        1.6362E+02
200     0.003000538    3.0001E-03     x   x   1.6362E+02        1.6362E+02
300     0.003000808    3.0002E-03     x   x   1.6361E+02        1.6361E+02
400     0.003001077    3.0002E-03     x   x   1.6360E+02        1.6360E+02
500     0.003001346    3.0003E-03     x   x   1.6359E+02        1.6359E+02
600     0.003001615    3.0004E-03     x   x   1.6359E+02        1.6359E+02
700     0.003001884    3.0004E-03     x   x   1.6358E+02        1.6358E+02
800     0.003002153    3.0005E-03     x   x   1.6357E+02        1.6357E+02
900     0.003002423    3.0005E-03     x   x   1.6356E+02        1.6356E+02
1000    0.003002692    3.0006E-03     x   x   1.6356E+02        1.6356E+02
1100    0.003002961    3.0007E-03     x   x   1.6355E+02        1.6355E+02
1200    0.00300323     3.0007E-03     x   x   1.6354E+02        1.6354E+02
1300    0.003003499    3.0008E-03     x   x   1.6353E+02        1.6353E+02
1400    0.003003768    3.0008E-03     x   x   1.6353E+02        1.6353E+02
1500    0.003004038    3.0009E-03     x   x   1.6352E+02        1.6352E+02
1600    0.003004307    3.0010E-03     x   x   1.6351E+02        1.6351E+02
1700    0.003004576    3.0010E-03     x   x   1.6350E+02        1.6350E+02
1800    0.003004845    3.0011E-03     x   x   1.6350E+02        1.6350E+02
1900    0.003005114    3.0011E-03     x   x   1.6349E+02        1.6349E+02
2000    0.003005383    3.0012E-03     x   x   1.6348E+02        1.6348E+02
2100    0.003005653    3.0012E-03     x   x   1.6347E+02        1.6347E+02
2200    0.003005922    3.0013E-03     x   x   1.6347E+02        1.6347E+02
2300    0.003006191    3.0014E-03     x   x   1.6346E+02        1.6346E+02
2400    0.00300646     3.0014E-03     x   x   1.6345E+02        1.6345E+02
```

[Close] [Export]

SMART|DT – Analysis Information (Growth)

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Analysis
- Output Options
- Growth
- Probabilistic

Model: Master Curve
Source: User Generated

Master Curve

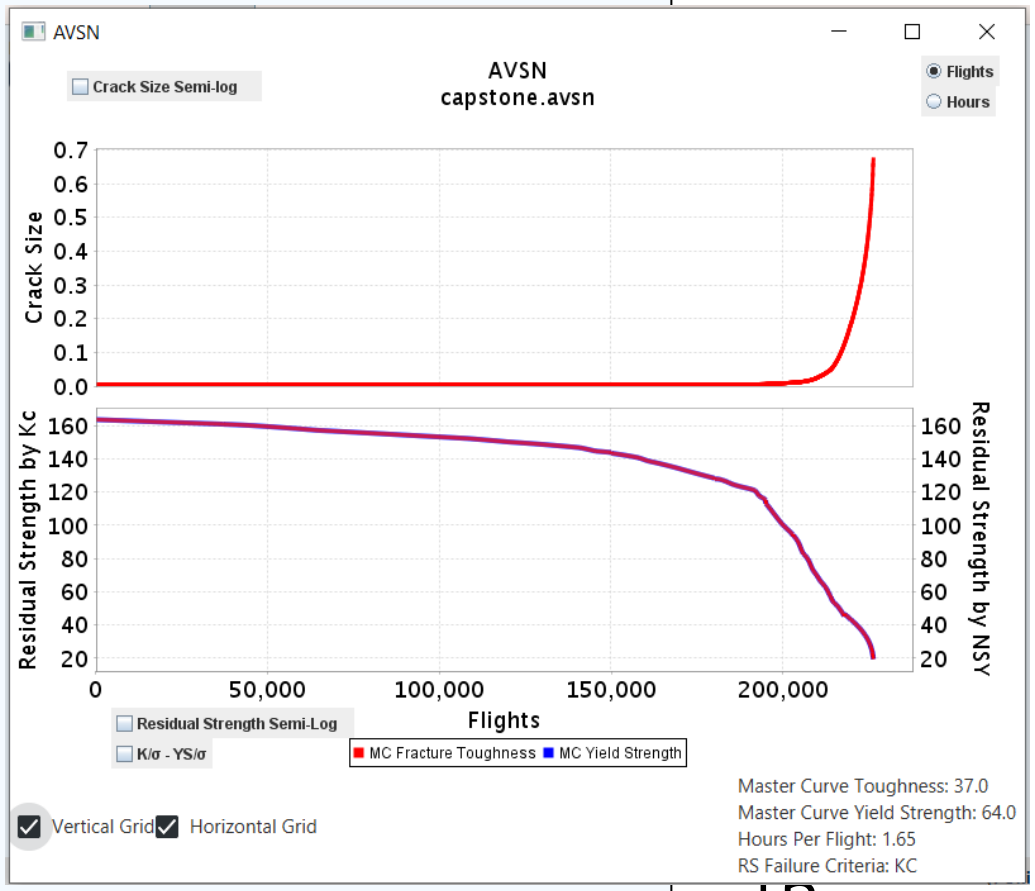
AVSN FILE
capstone1.avsn | Browse | View Text | Plot

MASTER CURVE FRACTURE TOUGHNESS
37.0

MASTER CURVE YIELD STRENGTH
64.0

HOURS PER FLIGHT
1.65

FAILURE CRITERIA
KC



SMART|DT – Analysis Information (Probabilistic)

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Analysis

- Output Options
- Growth
- Probabilistic

Method

Adaptive Importance Sampling

- Monte Carlo
- Adaptive Importance Sampling

Random Seed

2394

0.1

Samples Per Iteration

100

Maximum Iterations

100

SMART|DT – Material Information

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Category	Group	Treatment	Form, Orientation
Custom	2014 Series	7475-T7351	Plate L
Aluminum	2024 Series	7475-T761	Plate LT
Steel	2124 Series	7475-T7651	Plate TL
Titanium	2224 Series		
	7050 Series		
	7075 Series		
	7150 Series		
	7175 Series		
	7475 Series		

Summary

Length: Inches
Stress: KSI

Category: Aluminum
Group: 7475 Series
Treatment: 7475-T7351
Form, Orientation: Plate TL

FRACTURE TOUGHNESS

T = 1.3-4.0

DISTRIBUTION: Normal

MEAN: 37.0

STDEV: 3.8

YIELD STRENGTH

DISTRIBUTION: Deterministic

VALUE: 57.0

ULTIMATE STRENGTH

DISTRIBUTION: Deterministic

VALUE: 70.0

PARIS CONSTANT Log(C)

DISTRIBUTION: Deterministic

VALUE: 0.0

PARIS EXPONENT

DISTRIBUTION: Deterministic

VALUE: 0.0

SMART|DT – Geometry Information

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Equivalent Initial Flaw Size (EIFS)

Category	Group	Data Set	Summary
Custom			
Commercial Transport			
Military Fighter			
Military Transport			

Initial Crack Size Distribution

DISTRIBUTION

LogNormal

MEAN 0.009055 **STDEV** 0.001252

Aspect Ratio

DISTRIBUTION

Deterministic

VALUE 1.0

The EIFS is traditionally determined through the process of growing in-service or tear-down cracks backwards to time zero. As such, the results are dependent upon the aircraft location, assumed material parameters, and loading history. As a result, it is not recommended to use an EIFS distribution for a different application than for which it was derived. The EIFS values are provided here as a guide and care should be taken to select the distribution that best matches the aircraft mission, joint geometry and manufacturing methods, or ensure that the distribution is appropriately conservative.

SMART|DT – Loading Information

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Extreme Value Distribution (EVD) Method

Ultimate Load

Limit / Ultimate Load

19.5

~~Analysis~~ Inspection Parameters

Inspection Parameter	Value
Number of Inspection Types	~~One~~ Single Repair
Inspection Type	Sliding Probe – Eddy Current
Inspection Schedule	17,300 Flights (28,500 Hr) Initial 650 Flights (1,100 Hr) Repeat
Probability of Inspection	90%
Probability of Detection	Deterministic
Detectable Crack Size	.20 inch
Repair Crack POD	.20 inch
Repair Crack Size	Mean = 0.009055 in Standard Deviation = 0.001252 in

SMART|DT – Inspection Information

SMART|DT Capstone.smdt

File Help

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Inspection Presets

Name
Inspection 1

Delete

Inspections

Flights

Delete

Add Inspection Preset

Add Inspection Preset

Name: Inspection 1

Material	Inspection Type	Geometry	Equipment	Summary
Custom				
Aluminum				

Probability of Detection

DISTRIBUTION: Deterministic_

LENGTH (c): 0.2

DEPTH (a): 0.0

Probability of Inspection

VALUE: 0.9

Repaired Crack Size

Same as Original

Custom

Perfect

DISTRIBUTION: LogNormal

MEAN: 0.009055

STANDARD DEVIATION: 0.001252

Cancel Add

Edit Add

SMART|DT – Inspection Information

SMART|DT Capstone.smdt

File Help

SMART|DT

Information Analysis Material Geometry Loading Inspections Run Results

Inspection Presets

Name	Type	Inspection Prob.	Detection Prob.	Repaired Crack
Inspection 1		0.9	2c: 0.2 a: 0.0	* μ0.009055 σ0.001252 LN

Delete Edit Add

Add Inspections

Add Inspections

Preset

Inspection 1

Quantity	Minimum Flights	Maximum Flights	Frequency (Flights)
Multiple	17300	30300	650

Cancel Add

17300				
17950				
18600				
19250				
19900				
20550				
21200				
21850	Inspection 1	0.9	2c: 0.2 a: 0.0	* μ0.009055 σ0.001252
22500	Inspection 1	0.9	2c: 0.2 a: 0.0	* μ0.009055 σ0.001252

Delete Edit Add

SMART|DT – Run

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Ready to Start | Start Analysis

DAT File

```
! ------------------------------------------------------------------------
!        AIRCRAFT INFORMATION
! ------------------------------------------------------------------------
TITLE = Capstone
AC_MAKE = Acme
AC_MODEL = Sky Runner
AC_SERIAL_NUM = All
AC_TCDS = TCDSSR1
! ------------------------------------------------------------------------
!        METHOD
! ------------------------------------------------------------------------
INTEGRATION_METHOD = AIS  2394
AIS_TARGET_COV = 0.1
AIS_NSAMPLES = 100
AIS_MAXITER  = 100
POF_MAX_INC = 30000 50
ANALYSIS_TIME_UNITS = Flights
```

Analysis Details

Show/Export

SMART|DT – Run

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

0% complete. Start Analysis

DAT File

```
! ------------------------------------------------------------------------
!       AIRCRAFT INFORMATION
! ------------------------------------------------------------------------
TITLE = Capstone
AC_MAKE = Acme
AC_MODEL = Sky Runner
AC_SERIAL_NUM = All
AC_TCDS = TCDSSR1
! ------------------------------------------------------------------------
!       METHOD
! ------------------------------------------------------------------------
INTEGRATION_METHOD = AIS  2394
AIS_TARGET_COV = 0.1
AIS_NSAMPLES = 100
AIS_MAXITER  = 100
POF_MAX_INC = 30000 50
ANALYSIS_TIME_UNITS = Flights
```

Analysis Details

```
      40 % complete.
      50 % complete.
      60 % complete.
      70 % complete.
      80 % complete.
      90 % complete.
     100 % complete.

*********************************************
********* PDTA analysis complete ************
*********************************************

Total CPU  time =     33.140 secs
Total wall time =      5.166 secs
```

Show/Export

Total Runtime: 5 seconds

Master Curve .out File

SMart|DT - OUT content

```
------------------------------------------------------------------------------------------------------------------------

                                                FRACTURE MECHANICS

------------------------------------------------------------------------------------------------------------------------
Crack growth code:                                    MASTERC_USER
Master Curve file:                                    capstone1.avsn
Master Curve Fracture Toughness                      37.0000
Master Curve Yield Strength                          64.0000
Hours Per Flight                                      1.6500

Failure criterion:                                       KC

Initial crack size distribution:                      LOGNORMAL
Initial crack size mean (X):                          0.0091
Initial crack size standard deviation (X):            0.0013

Fracture toughness distribution:                      NORMAL
Fracture toughness mean:                             37.0000
Fracture toughness standard deviation:                3.8000

------------------------------------------------------------------------------------------------------------------------

                                               LOADING AND EVD PARAMETERS

------------------------------------------------------------------------------------------------------------------------
EVD PARAMETERS

EVD value distribution:                               LIMIT
EVD location:                                        19.5000
 A deterministic limit load is used with value:  19.5000


USAGE PARAMETERS

Number of Usages:                                        0

------------------------------------------------------------------------------------------------------------------------

                                                INSPECTIONS

------------------------------------------------------------------------------------------------------------------------
 Inspections: Time [Type]:
                                                17300 [ 1]  17950 [ 1]  18600 [ 1]  19250 [ 1]  19900
```

Close Export

Master Curve .out File

SMart|DT - OUT content

SUMMARY OF OUTPUT DATA

POF results in CSV format were written to Capstone_pof.csv

Flight Number	RESULTS WITHOUT INSPECTIONS POF	Cumulative	RESULTS WITH INSPECTIONS POF	Cumulative	
0	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
50	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
100	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
150	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
200	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
250	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
300	0.000E+000	0.000E+000	0.000E+000	0.000E+000	
29350	9.608E-001	1.000E+000	1.360E-007	1.539E-003	
29400	9.629E-001	1.000E+000	1.360E-007	1.546E-003	
29450	9.649E-001	1.000E+000	1.360E-007	1.553E-003	
29500	9.668E-001	1.000E+000	1.360E-007	1.560E-003	
29550	9.686E-001	1.000E+000	1.360E-007	1.567E-003	
29600	9.703E-001	1.000E+000	1.360E-007	1.573E-003	
29650	9.720E-001	1.000E+000	1.360E-007	1.580E-003	
29650	9.720E-001	1.000E+000	1.360E-007	1.580E-003	<--Insp
29700	9.735E-001	1.000E+000	1.360E-007	1.587E-003	
29750	9.750E-001	1.000E+000	1.360E-007	1.594E-003	
29800	9.764E-001	1.000E+000	1.364E-007	1.601E-003	

Close Export

Master Curve .out File

SMartJDT - OUT content

SUMMARY OF THE EFFECTS OF INSPECTIONS ON THE POF

----------RESULTS WITH INSPECTIONS----------

FLT NUM	POF	CTPOF	Pct_Cracks_Detected
17300	8.400E-008	6.899E-005	
17300	1.830E-008	6.899E-005	13.61
17950	4.880E-008	8.812E-005	
17950	1.739E-008	8.812E-005	8.79
18600	6.849E-008	1.102E-004	
18600	3.550E-008	1.102E-004	9.59
19250	1.154E-007	1.502E-004	
19250	7.858E-008	1.502E-004	10.56
19900	1.525E-007	2.182E-004	
19900	1.234E-007	2.182E-004	11.10
20550	1.592E-007	3.074E-004	
20550	1.373E-007	3.074E-004	11.01
21200	1.542E-007	4.013E-004	
21200	1.441E-007	4.013E-004	10.20
21850	1.611E-007	5.001E-004	
21850	1.439E-007	5.001E-004	8.67

Close Export

SMART|DT – Results

SMART|DT

Information | Analysis | Material | Geometry | Loading | Inspections | Run | Results

Results

Probabiliy of Failure

Load External POF

POF ☐ Cumulative

Probability of Failure (POF) vs. Flights

Flights ○ Hours

Probability of Failure (POF): 10^{0}, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}, 10^{-11}, 10^{-12}, 10^{-13}, 10^{-14}, 10^{-15}

0 | 5,000 | 10,000 | 15,000 | 20,000 | 25,000 | 30,000

Flights

POF (w/o Insp.) Capstone_pof.csv — POF (w/ Insp.) Capstone_pof.csv

Vertical Grid ☑ Horizontal Grid ☑

Initial Flaw Size
Lognormal
μ = .009055
σ = .001252

Fracture Toughness
μ = 37.0
σ = 3.8

Initial Inspection
28,500 Hours
Repeat Inspection
1,100 Hours

29

Questions