Introduction to Probabilistic Methods with Applications to Probabilistic Damage Tolerance Analysis

Adaptive Multiple Importance Sampling

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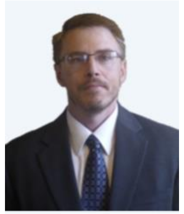
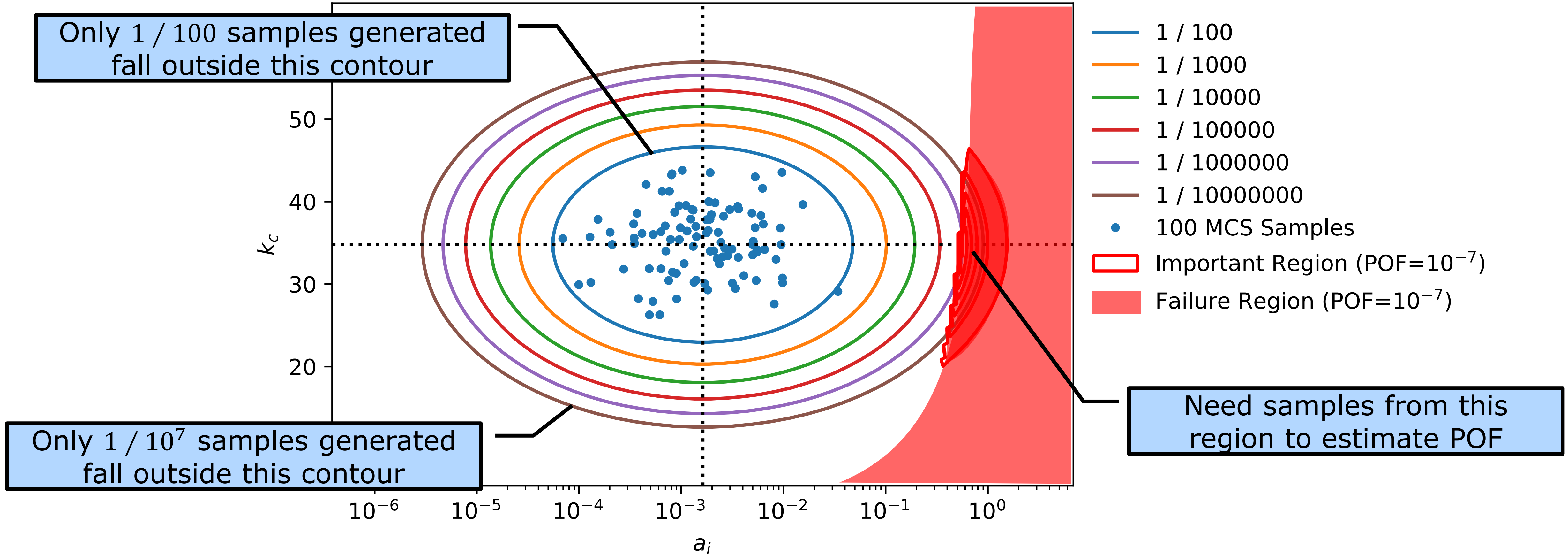


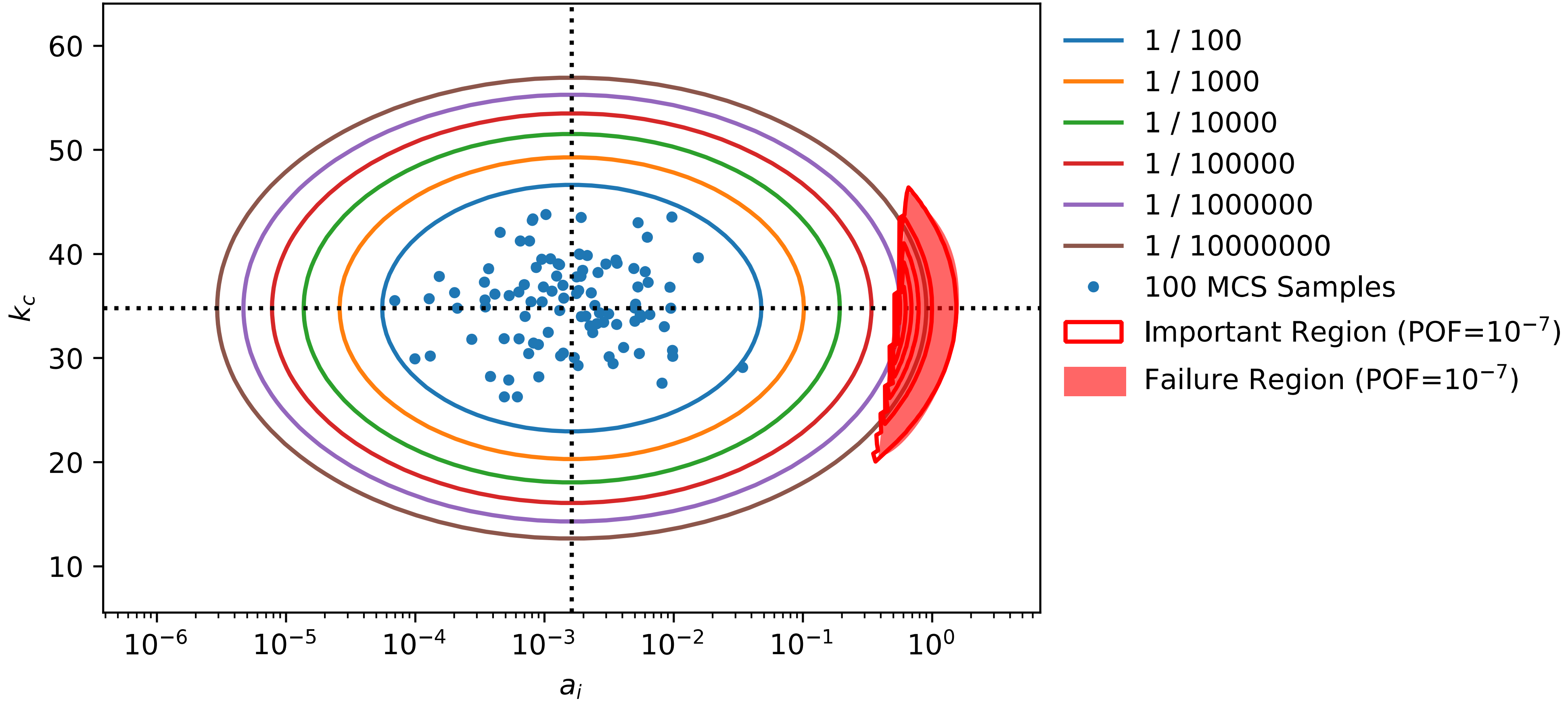

August 29, 2022

Monte Carlo Sampling – Limit State In 2-Dimensions



- *Standard Monte Carlo* (SMC) is simple and robust, but inefficient for estimating rare event probabilities
  - $N = \left(1 - \overline{P}\right) / \left(\overline{P}\,\delta_{\overline{P}}^{2}\right)$, so for $\overline{P} = 10^{-7}$ and $\delta_{\overline{P}} = 0.1$, $N = 10^{9}$

Optimal Sampling Region



- Only a small portion of the failure region is important for calculating POF
- Call this the *important region*
  - as drawn the important region (in red) accounts for 99% of the POF

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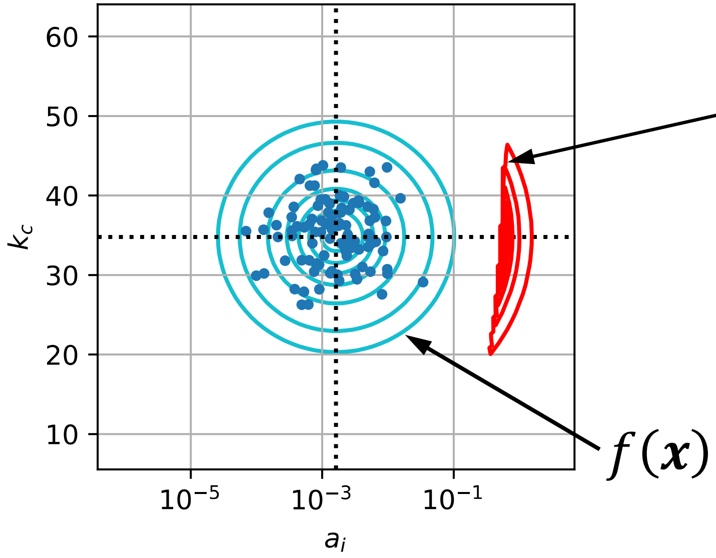
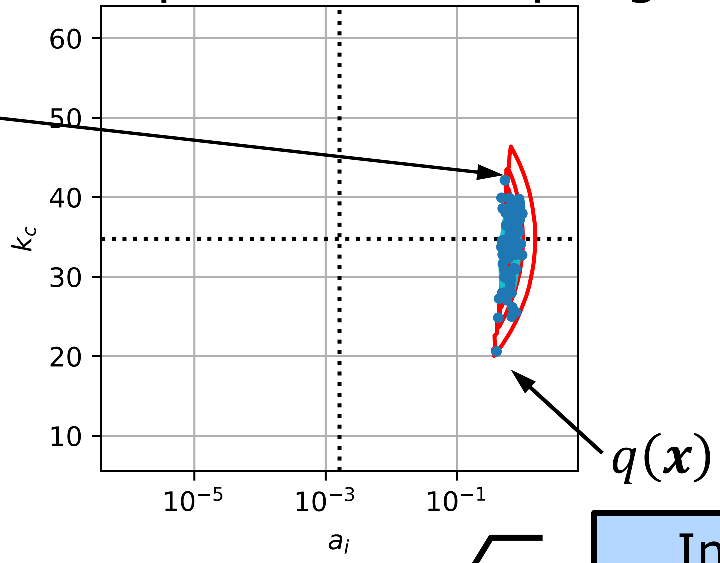
Importance Sampling

Standard Monte Carlo Sampling



region of importance

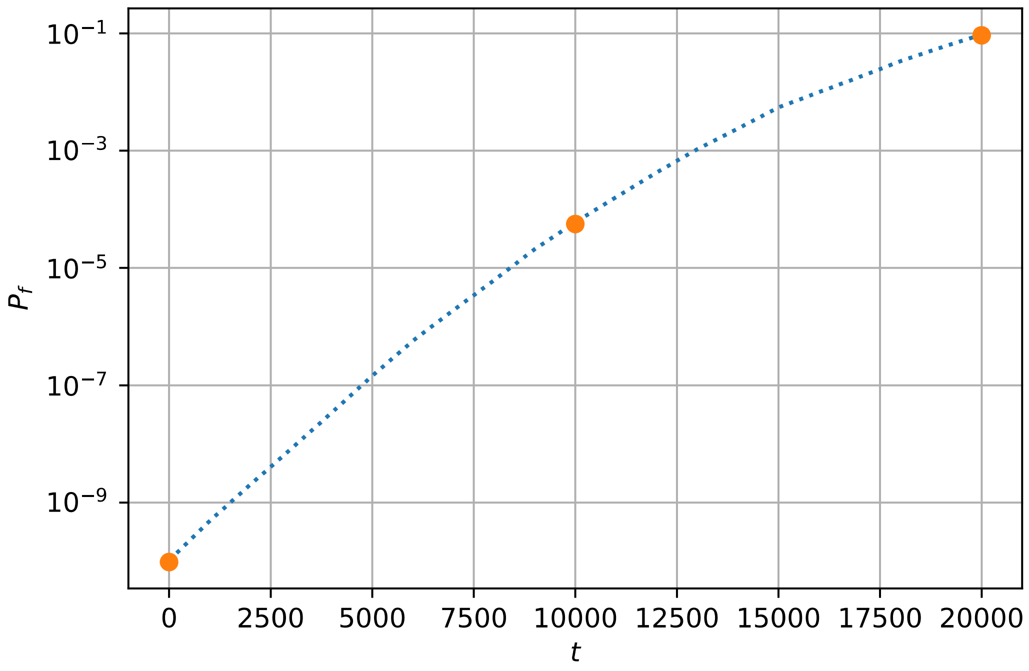
Importance Sampling

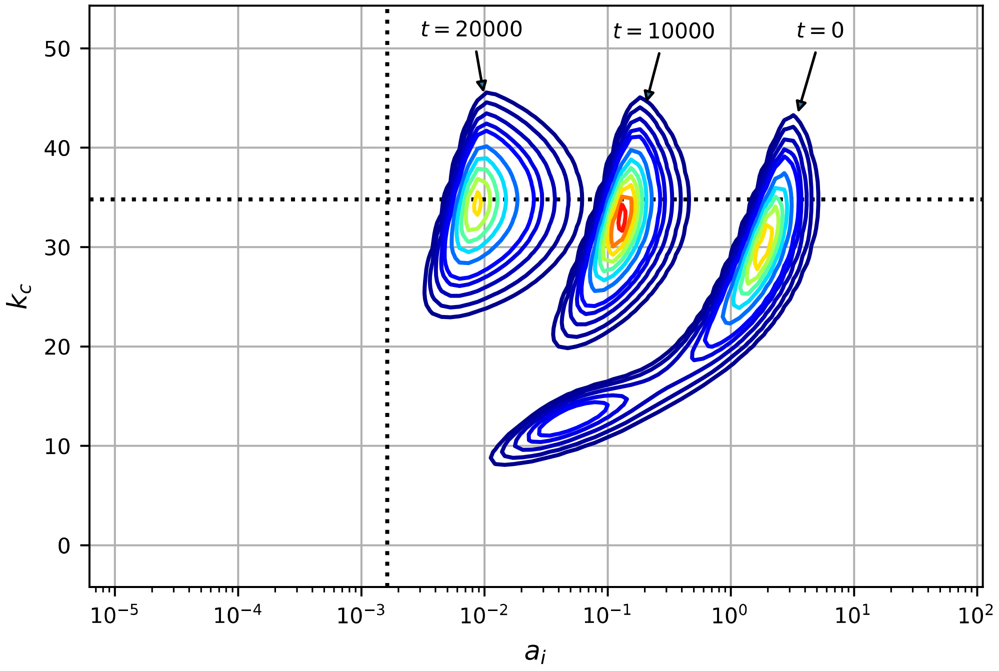


Importance weight
$w(\boldsymbol{x}) = f(\boldsymbol{x}) \,/\, q(\boldsymbol{x})$

$\mathbb{E}[H(\boldsymbol{x};t)] = \int H(\boldsymbol{x})\, f(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}$	$\mathbb{E}[H(\boldsymbol{x})] = \int H(\boldsymbol{x};t) \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})}\, g(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}$
$\hat{\mu} \approx \frac{1}{N} \sum_i H(\boldsymbol{x}_i;t)$	$\hat{\mu} \approx \frac{1}{N} \sum_i H(\boldsymbol{x}_i;t) \frac{f(\boldsymbol{x}_i)}{g(\boldsymbol{x}_i)}$
$\mathrm{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i (H(\boldsymbol{x}_i;t) - \hat{\mu})^2$	$\mathrm{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i \left(H(\boldsymbol{x};t) \frac{f(\boldsymbol{x}_i)}{g(\boldsymbol{x}_i)} - \hat{\mu} \right)^2$

Adaptive Importance Sampling





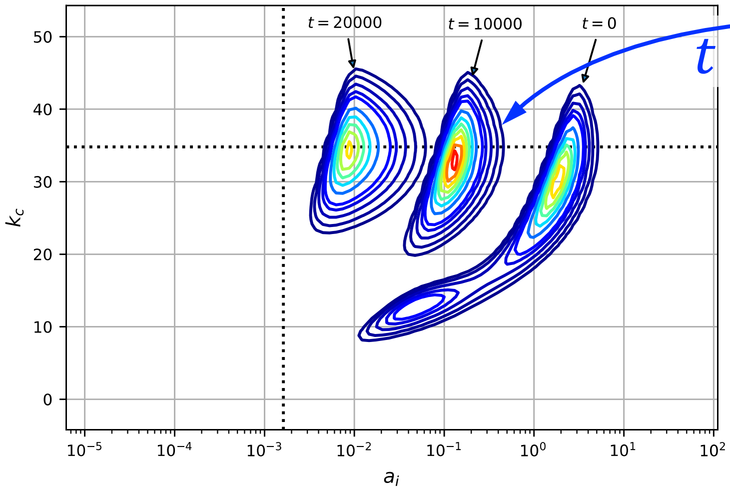
- PDTA requires evaluating POF over a range of times
- With traditional importance sampling, a sampling density must be adapted to the important region(s) for each evaluation time (and each inspection)
- The adaptation process is iterative, and becomes cumbersome when many important regions must be covered

Adaptive Multiple Importance Sampling Approach for PDTA

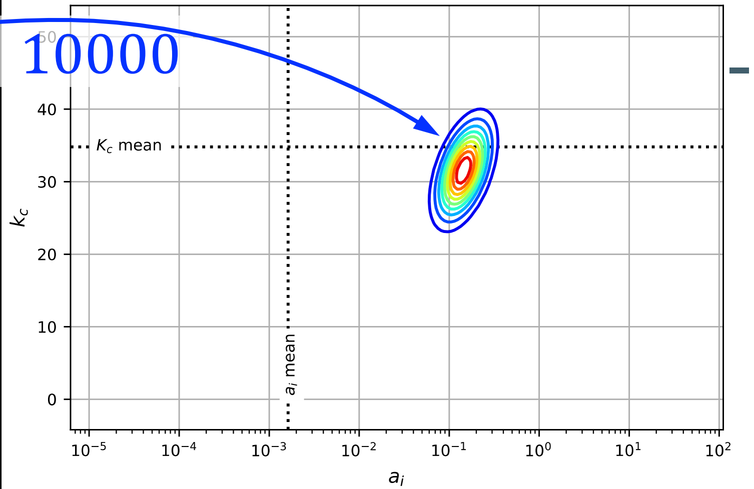
Important Region(s)

Adapted Sampling Region

Individual Times

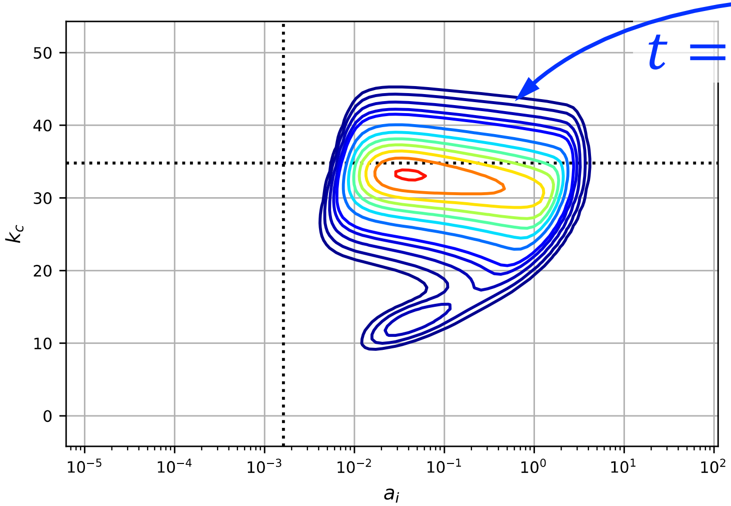


$t = 10000$

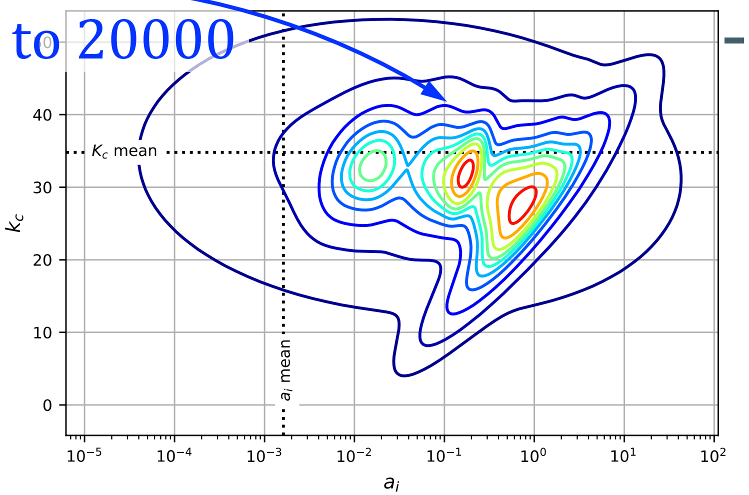


- Basic Importance sampling
  - Adapt individual sampling density for each evaluation time

Multiple Times



$t = 0$ to 20000



- Multiple Importance Sampling
  - Adapt a mixture density for the entire range of evaluation times

Going From Collection of Component Densities to Mixture Density

Importance weight for sample $\boldsymbol{x}_{ij}$

Nominal density

$$w_{mix}(\boldsymbol{x}_{ij}) = \sum_{m=1}^{N_{mix}} \boxed{\omega_m(\boldsymbol{x}_{ij})} \frac{f(\boldsymbol{x}_{ij})}{q(\boldsymbol{x}_{ij}, \boldsymbol{\theta}_j)} \quad \forall\ i, j$$

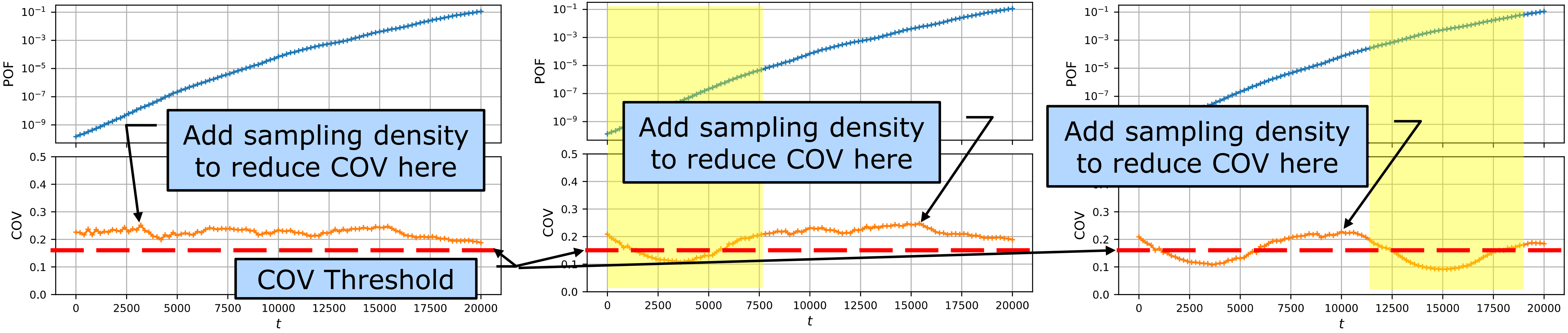
Mixture weights for sample $\boldsymbol{x}_{ij}$, $\sum_{m=1}^{N_{mix}} \omega_m(\boldsymbol{x}_{ij}) = 1$

j-th component density with parameters $\boldsymbol{\theta}_j$

$$\widehat{\text{POF}}\,(t) = \frac{1}{N} \sum_{i=1}^{N_{samp}} \sum_{j=1}^{N_{mix}} H(\boldsymbol{x}_{ij}; t)\, w_{mix}(\boldsymbol{x}_{ij})$$

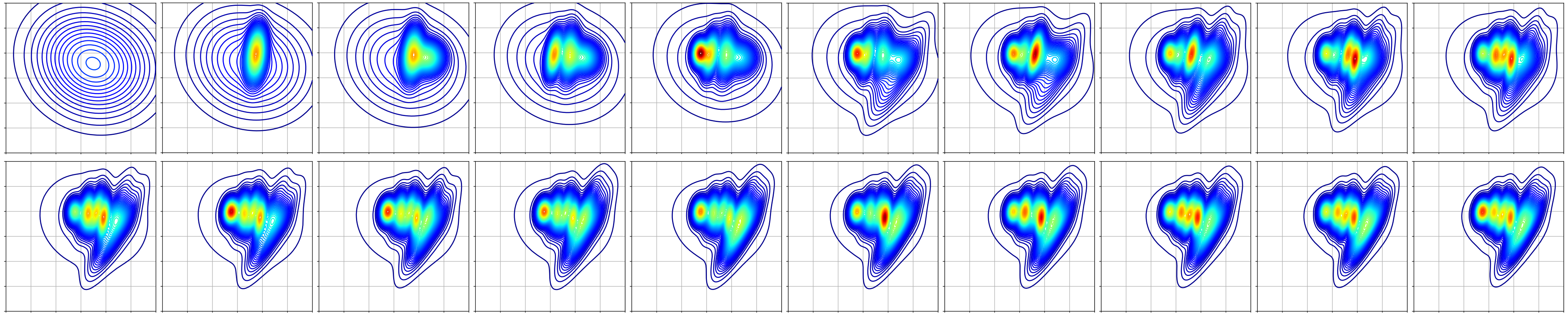
- The collection of individual sampling densities becomes a mixture density by calculating the importance weight of each sample as a weighted sum of the individual sampling density importance weights
- Adds complexity to importance weight calculation (and recalculation), but greatly reduces the number of sample crack growth evaluations

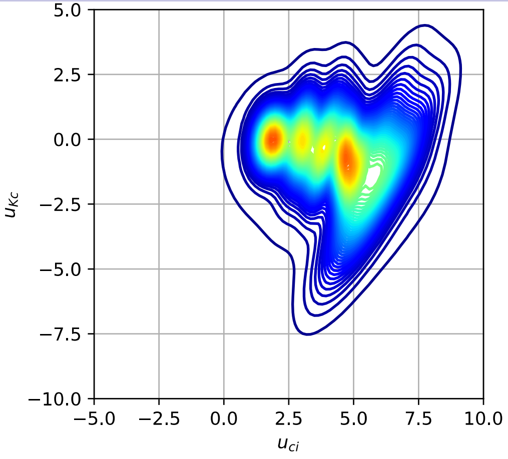
Mixture Density Adaptation



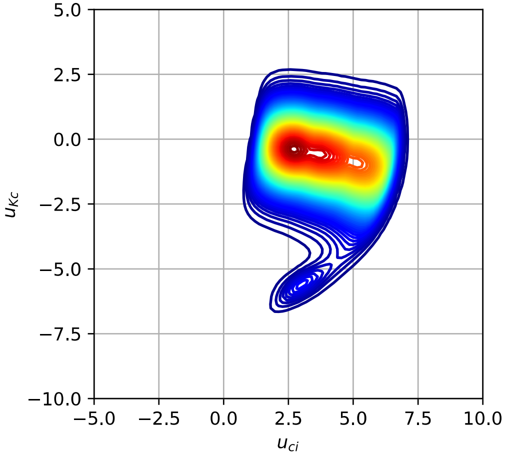
- Coefficient of Variation, $\mathrm{COV} = \widehat{POF} \Big/ \sqrt{\mathrm{Var}\left(\widehat{POF}\right)}$, is used to decide if each POF estimate is converged, and to pick which POF estimate (time value) to focus on with the next component density
- Adaptation continues iteratively until all POF estimate COVs are less than or equal to the COV threshold (user specified)

Mixture Density Adaptation

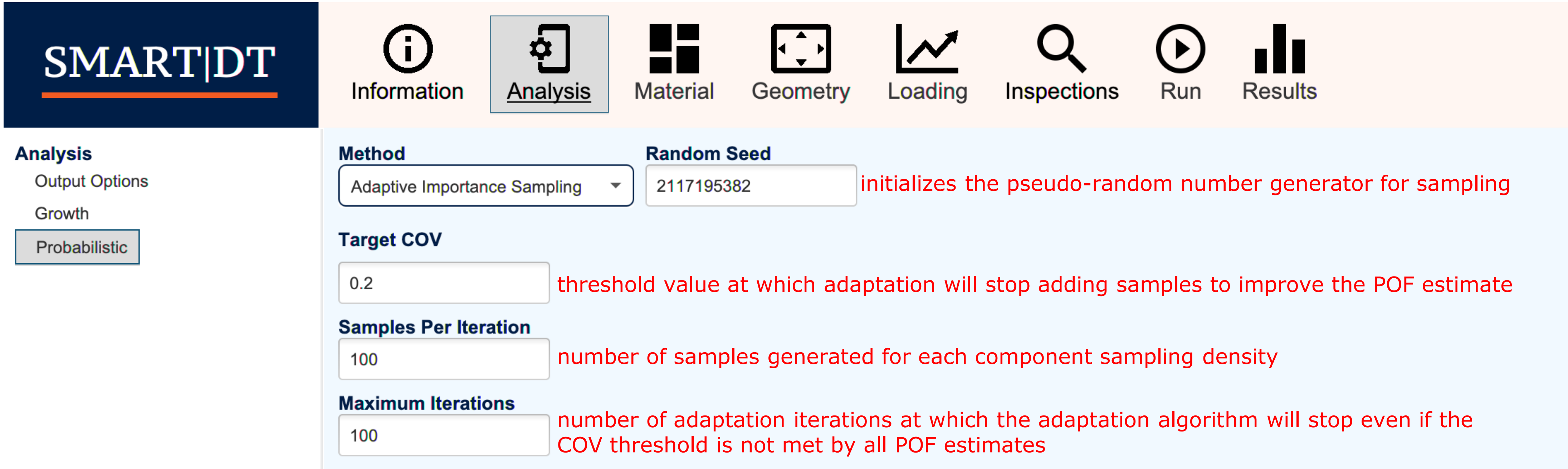




- Above: Progression of Mixture Density adaption - a multivariate normal component density, tailored to the POF(t) estimate with the highest COV, is added at each step
- Top Right: mixture density satisfying the COV threshold for all POF(t) estimates (1500 samples)
- Bottom Right: ideal sampling density (9000 samples – 300x300 grid)

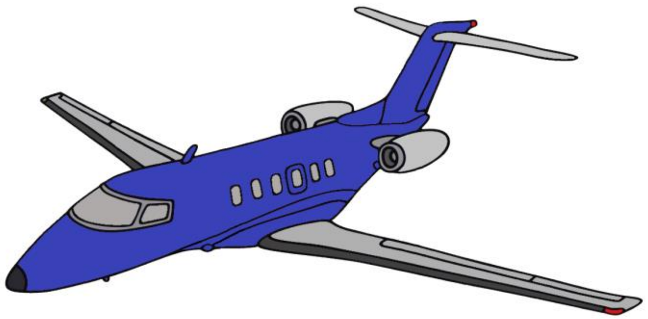


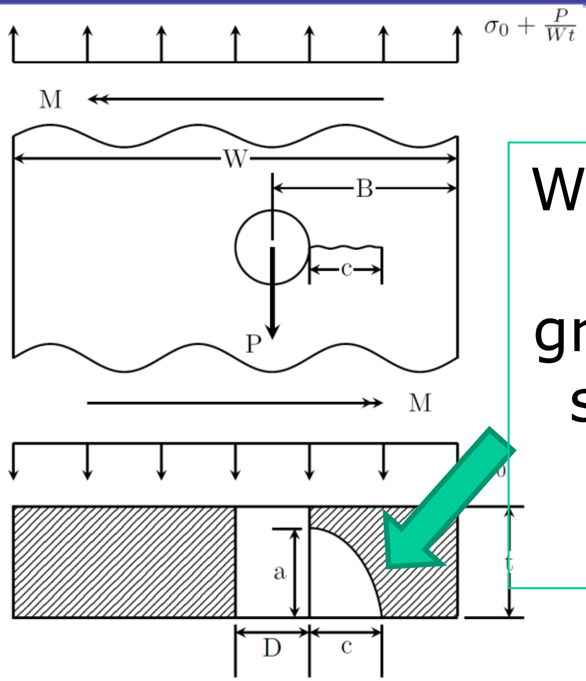
SMART|DT Adaptive Multiple Importance Sampling Setup



- To use Adaptive Multiple Importance Sampling in SMART|DT, go to the *Analysis* page (top navigation), select *Probabilistic* in the side navigation, and change the *Method* to Adaptive Importance Sampling

Adaptive Multiple Importance Sampling Example Problem



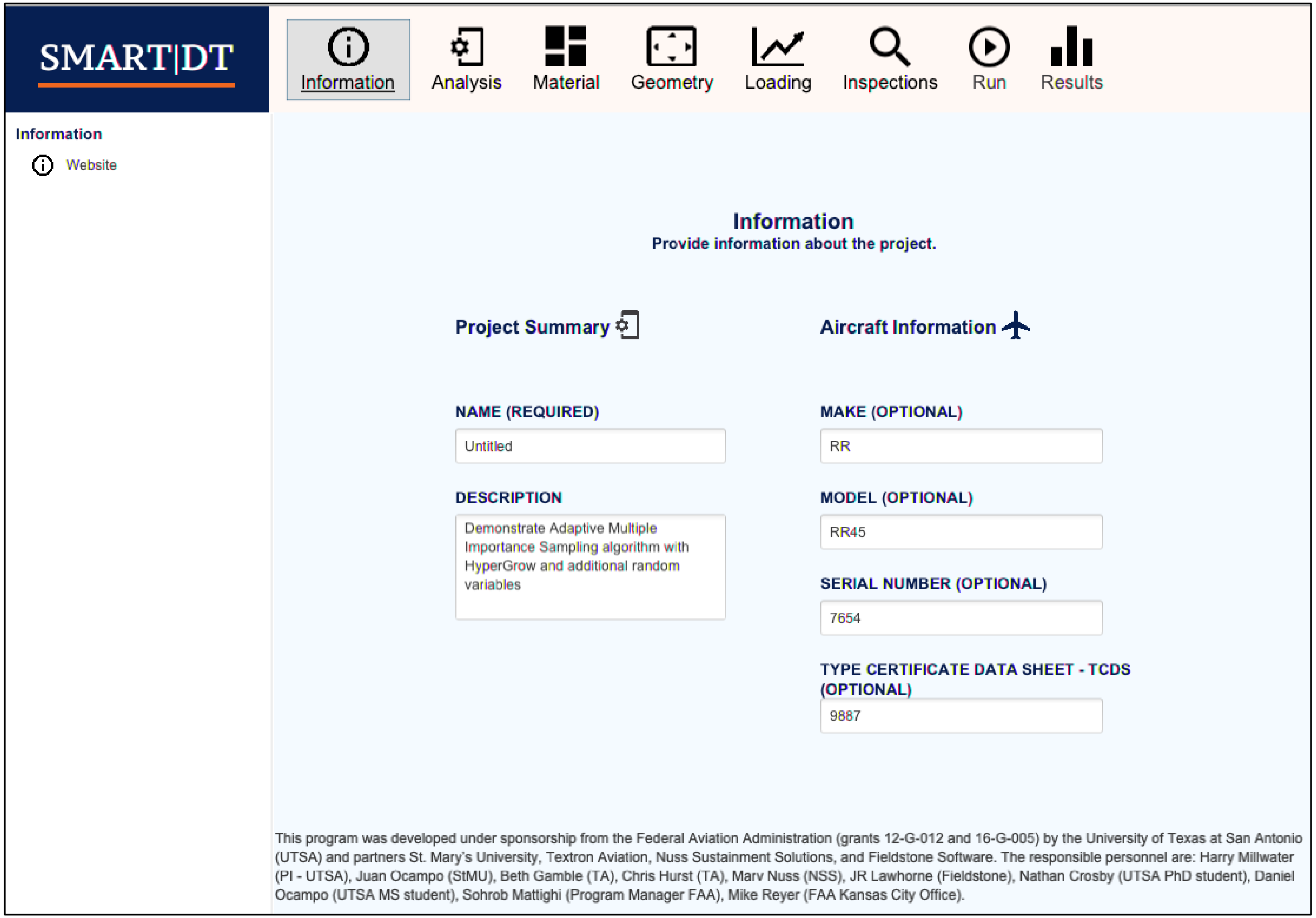


We will use HyperGrow to evaluate a crack growth curve for every sample with random da/dn, aspect ratio and hole diameter

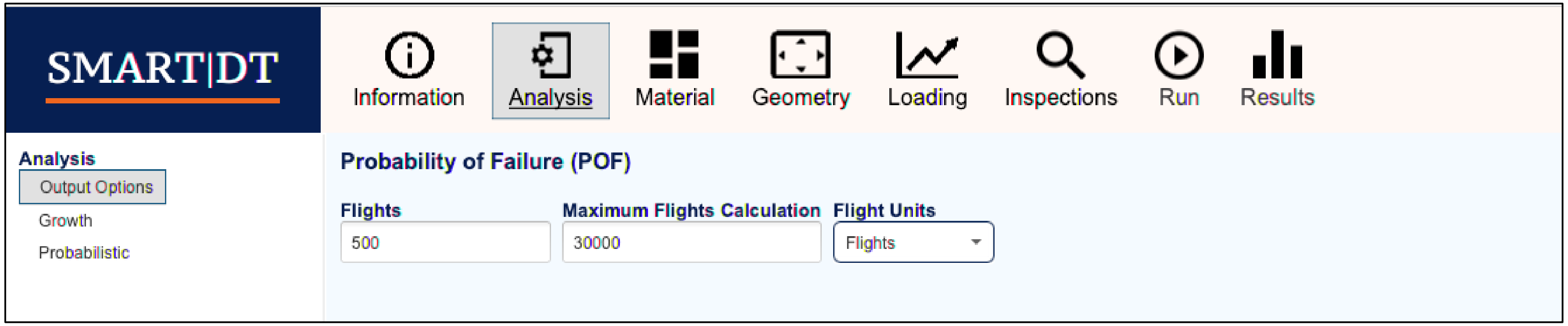
Geometric Variables	Value
Hole radius	0.164 in
Cap Thickness	0.175 in
Cap Width	1.80 in

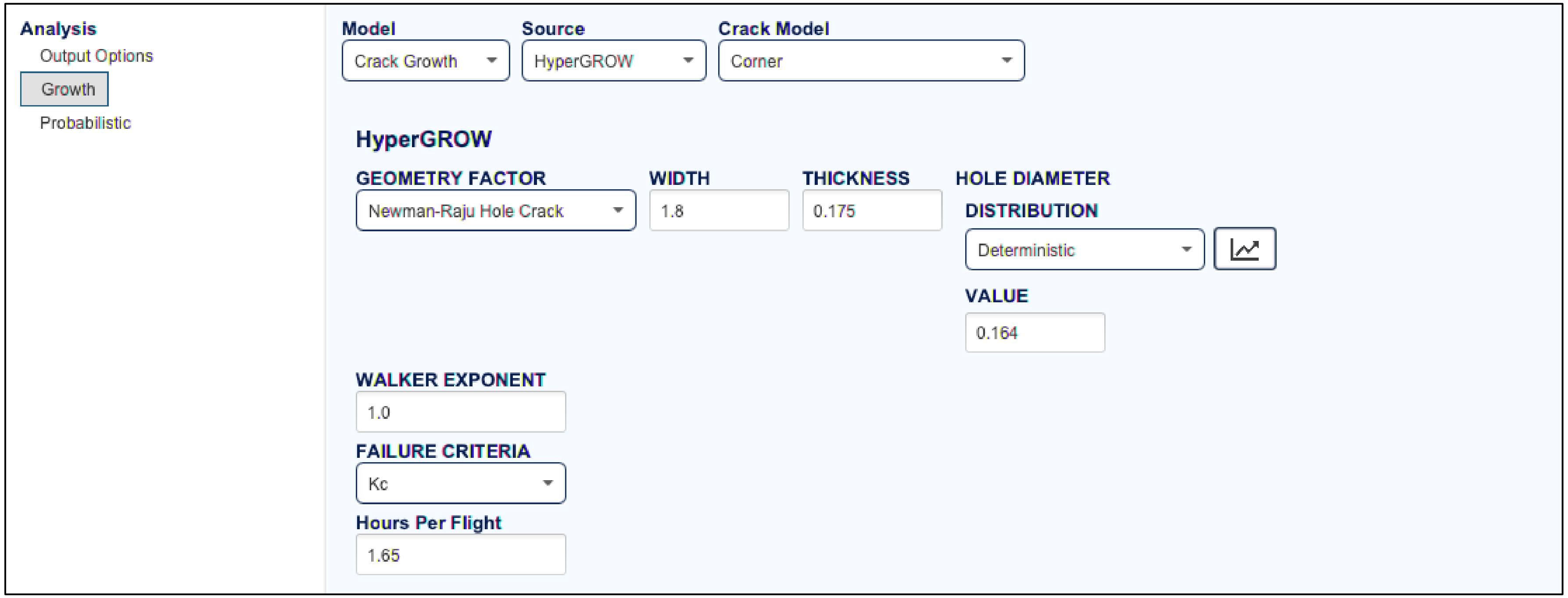
Hypergrow_NewmanRajuCorner.smdt

GUI - Information

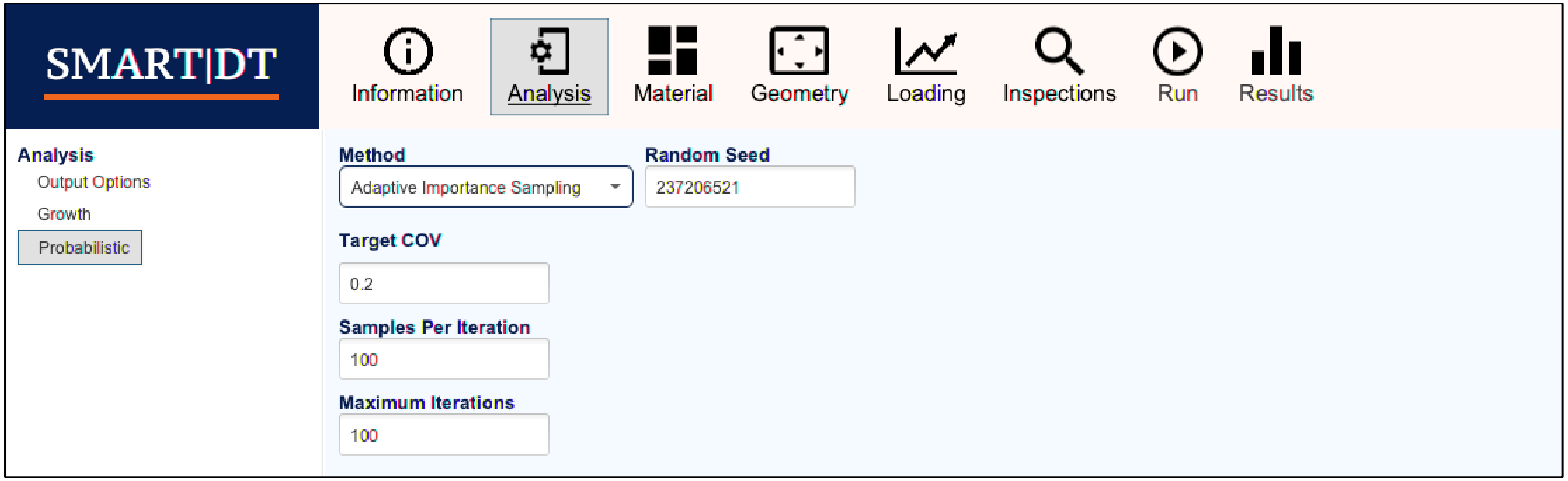


GUI – Analysis (I)

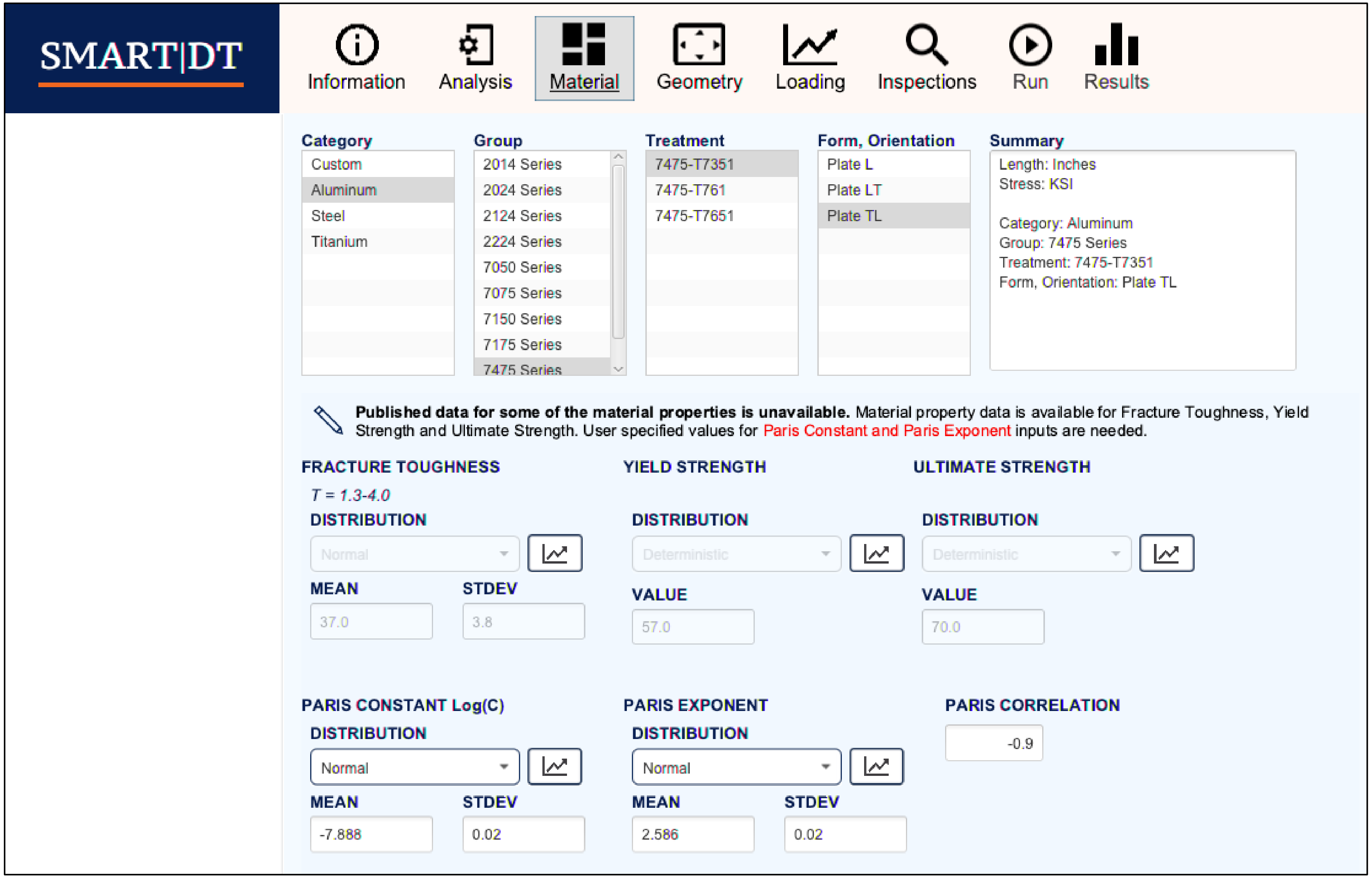




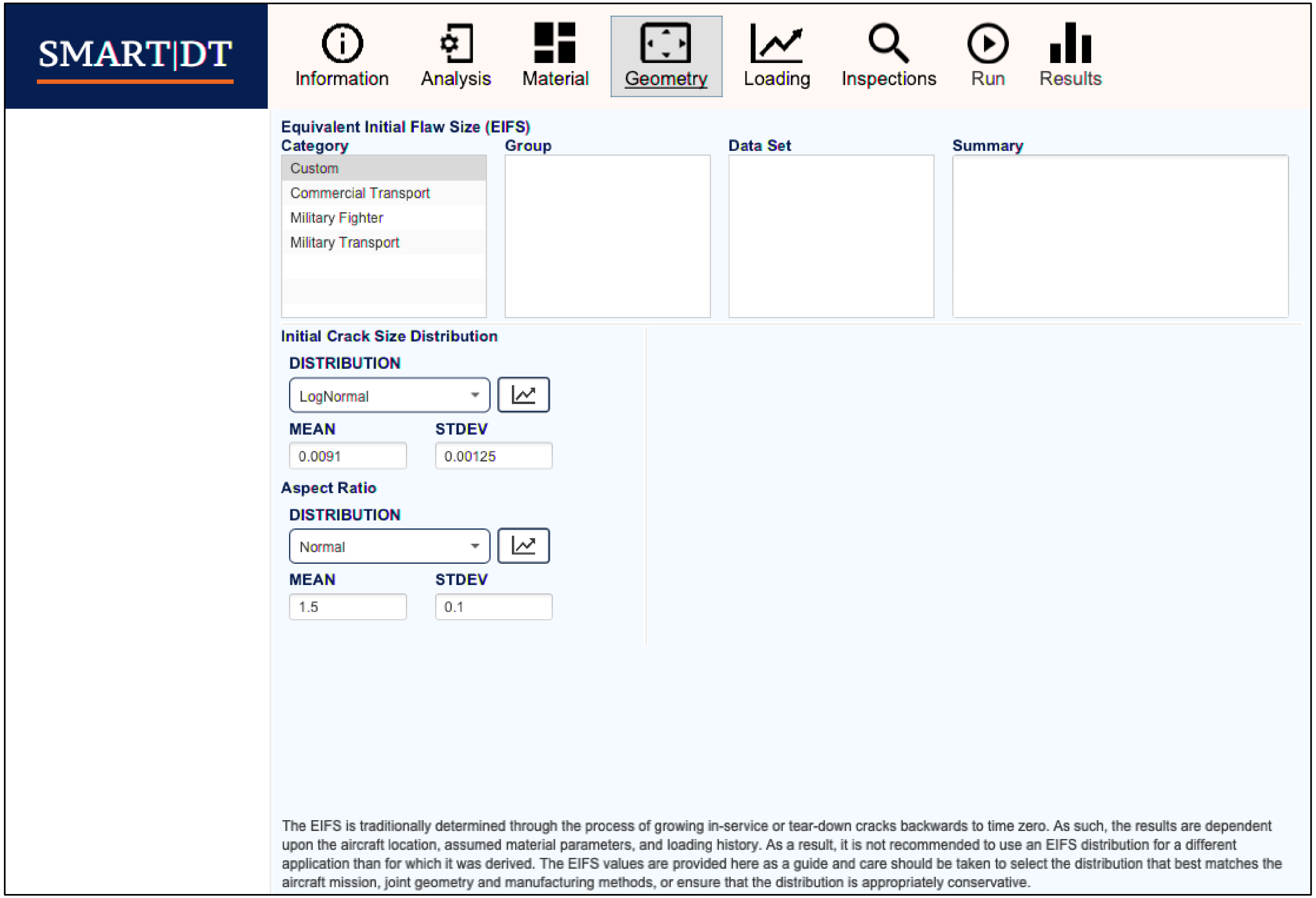
GUI – Analysis (II)



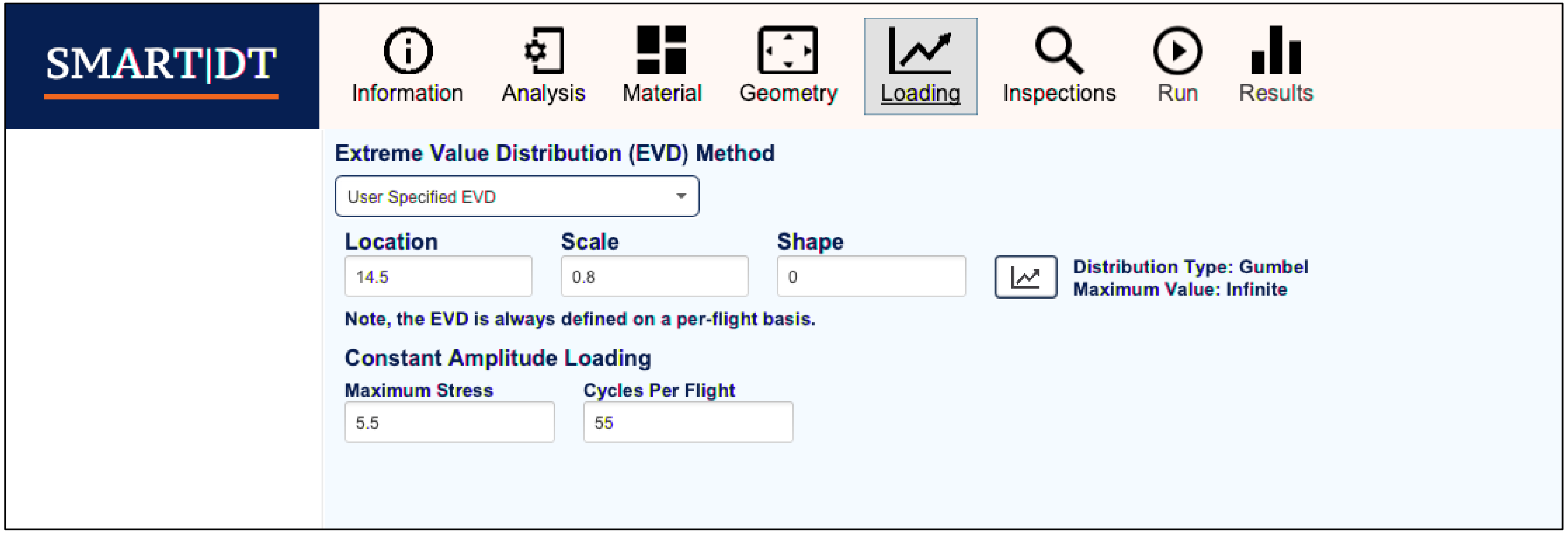
GUI - Material



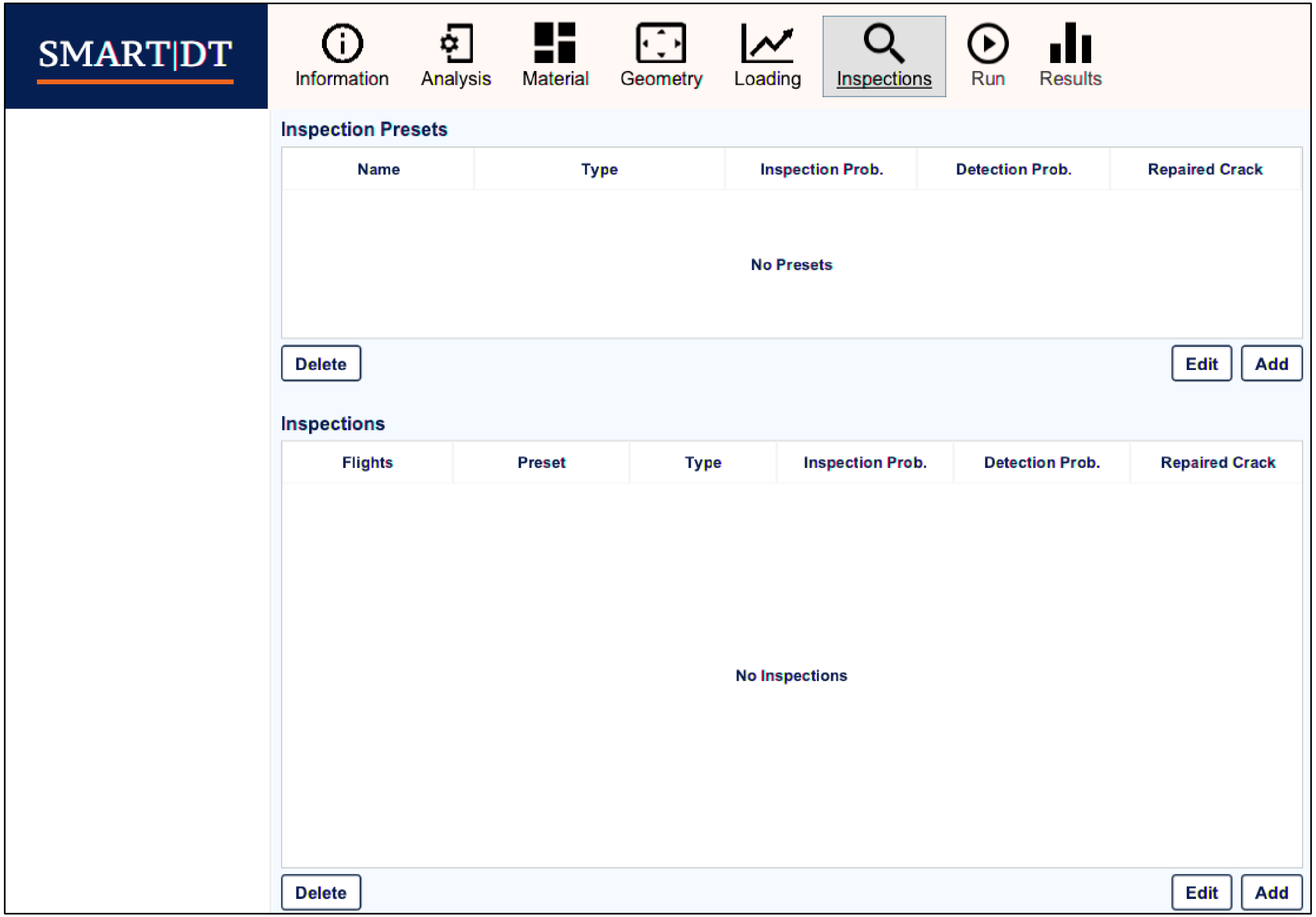
GUI – Geometry



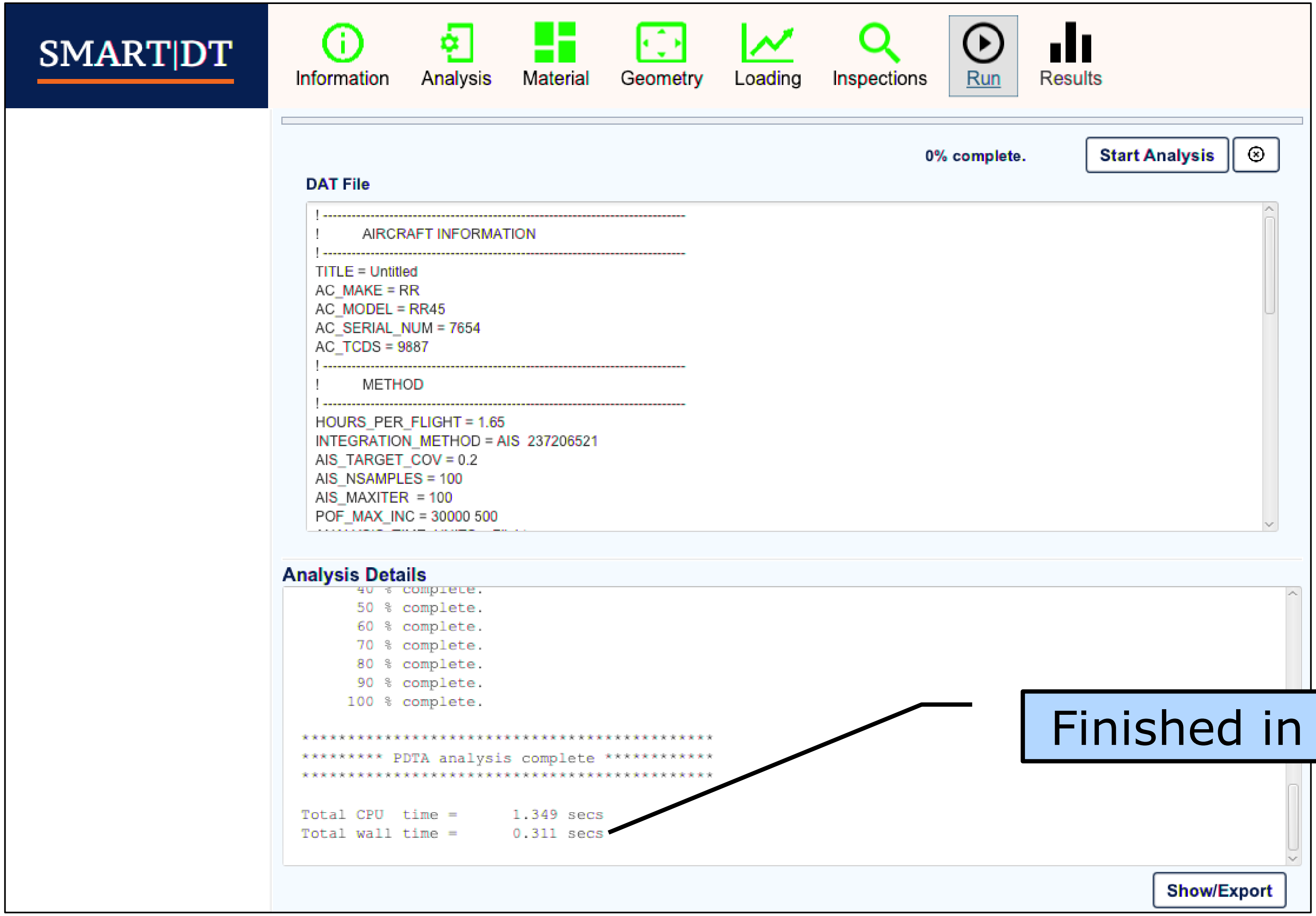
GUI - Loading


SMART|DT
Information
Analysis
Material
Geometry
Loading
Inspections
Run
Results
Extreme Value Distribution (EVD) Method
User Specified EVD
Location
14.5
Scale
0.8
Shape
0
Distribution Type: Gumbel
Maximum Value: Infinite
Note, the EVD is always defined on a per-flight basis.
Constant Amplitude Loading
Maximum Stress
5.5
Cycles Per Flight
55

GUI - Inspections

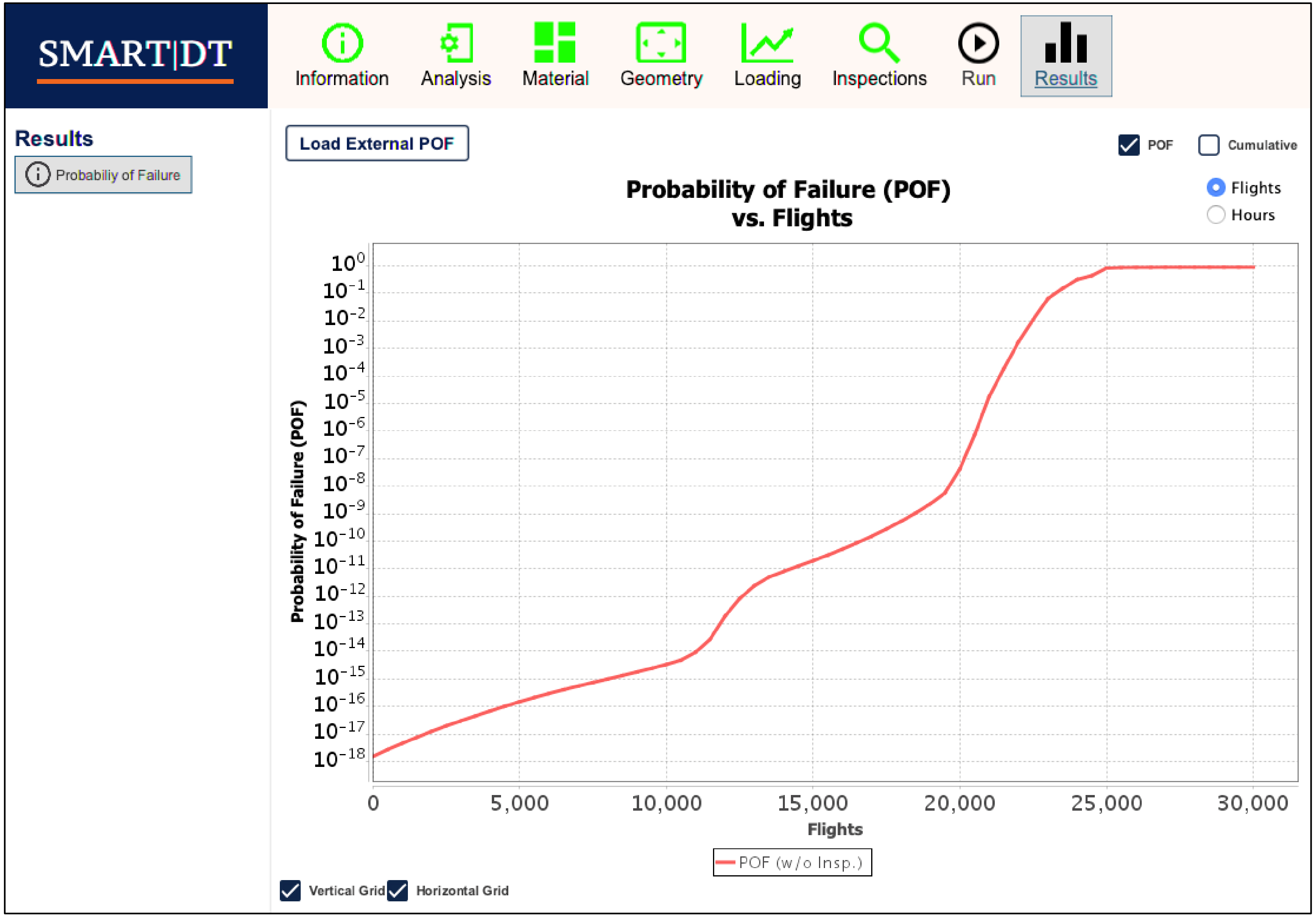


GUI - Run



Finished in 0.3 seconds

GUI - Results



SMART|DT
Information
Analysis
Material
Geometry
Loading
Inspections
Run
Results
Results
Probabiliy of Failure
Load External POF
POF
Cumulative
Probability of Failure (POF)
vs. Flights
Flights
Hours
Probability of Failure (POF)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
0
5,000
10,000
15,000
20,000
25,000
30,000
Flights
POF (w/o Insp.)
Vertical Grid
Horizontal Grid

Summary

- Introduce Adaptive Multiple Importance Sampling now available in SMART|DT
- This method reduces PDTA runtimes by 6 orders of magnitude compared to Standard Monte Carlo with 10^9 samples
- The speed of the method allows da/dn variation to be included in the analysis

Questions