



Introduction to Probabilistic Methods with Applications to Probabilistic Damage Tolerance Analysis



Adaptive Multiple Importance Sampling

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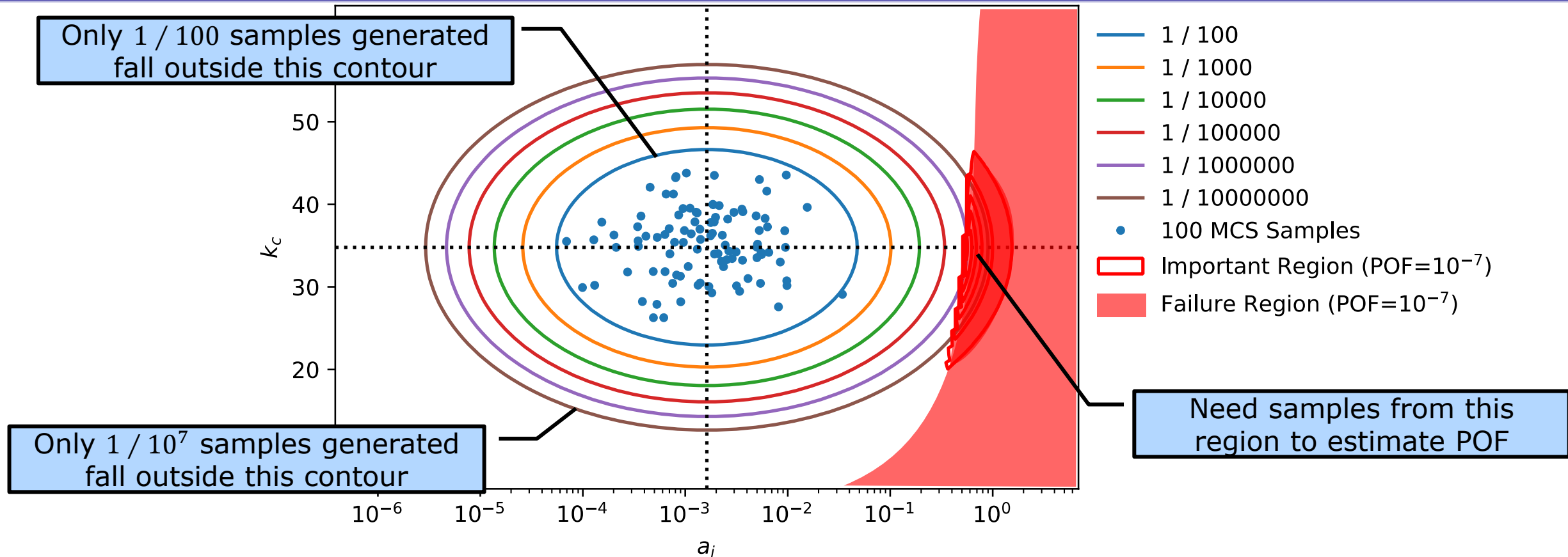
Marv Nuss, Nuss Sustainment Solutions



August 29, 2022



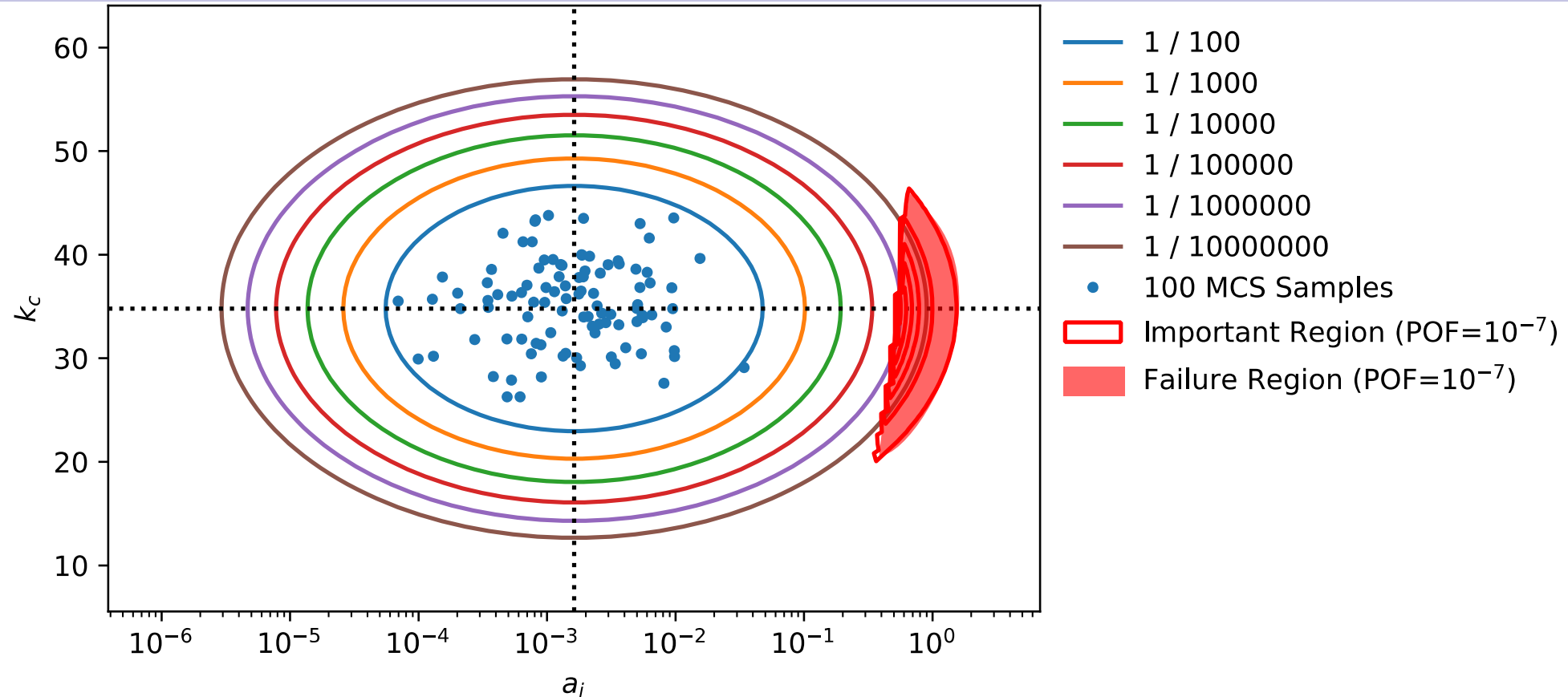
Monte Carlo Sampling – Limit State In 2-Dimensions



- *Standard Monte Carlo* (SMC) is simple and robust, but inefficient for estimating rare event probabilities
 - $N = (1 - \bar{P}) / (\bar{P} \delta_{\bar{P}}^2)$, so for $\bar{P} = 10^{-7}$ and $\delta_{\bar{P}} = 0.1$, $N = 10^9$



Optimal Sampling Region

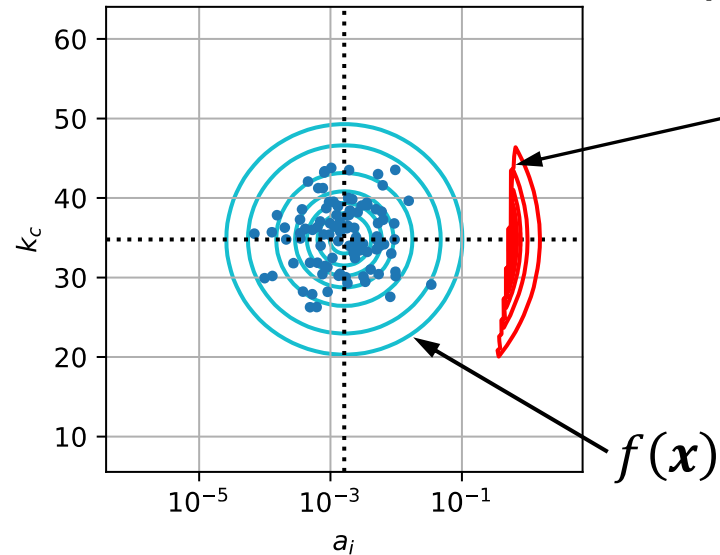


- Only a small portion of the failure region is important for calculating POF
- Call this the *important region*
 - as drawn the important region (in red) accounts for 99% of the POF

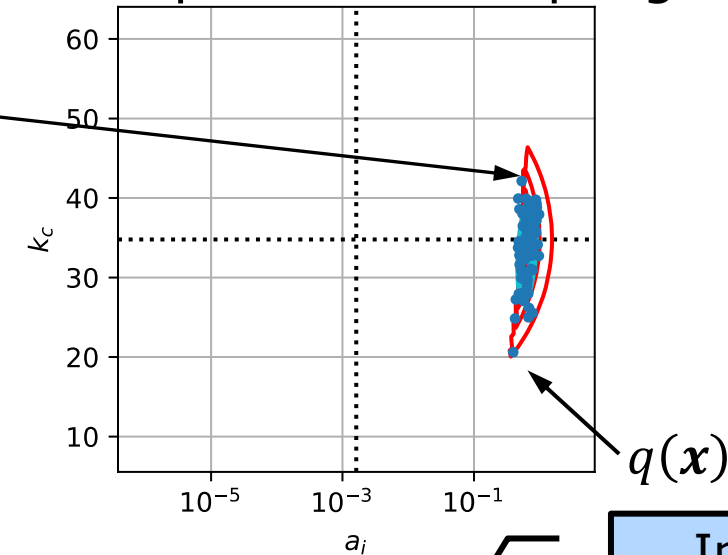


Importance Sampling

Standard Monte Carlo Sampling



Importance Sampling



Importance weight
 $w(x) = f(x) / q(x)$

$$\mathbb{E}[H(x; t)] = \int H(x) f(x) dx$$

$$\mathbb{E}[H(x)] = \int H(x; t) \frac{f(x)}{g(x)} g(x) dx$$

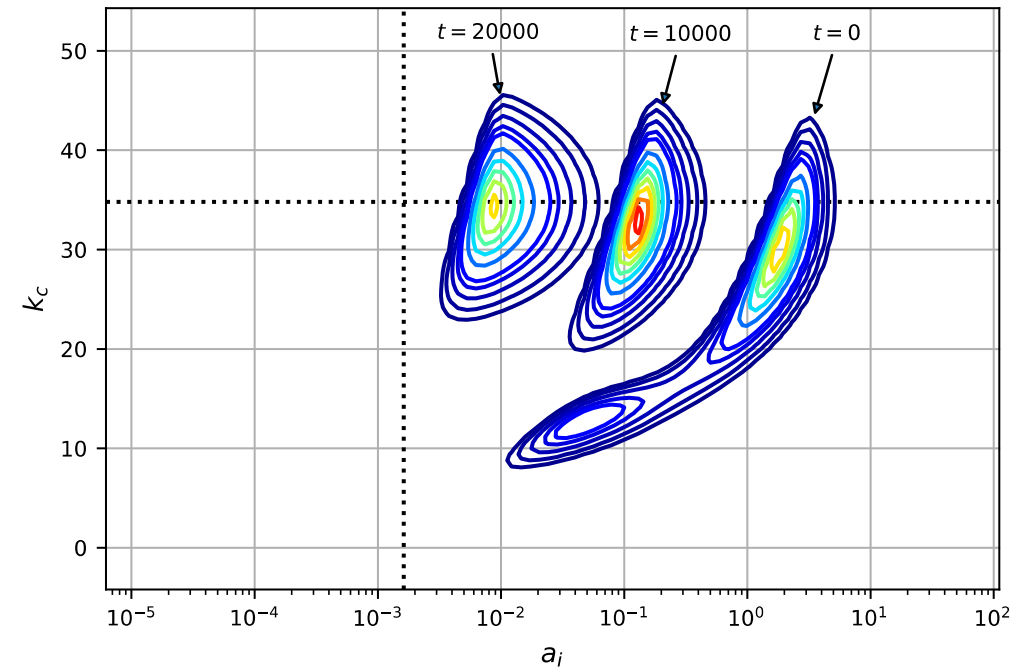
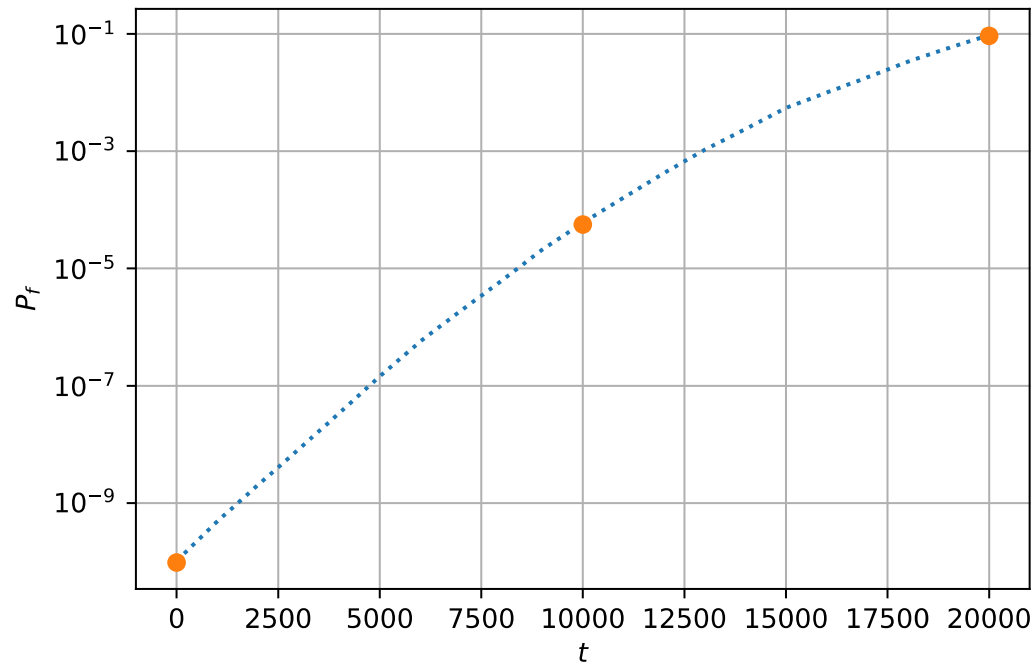
$$\hat{\mu} \approx \frac{1}{N} \sum_i H(x_i; t)$$

$$\hat{\mu} \approx \frac{1}{N} \sum_i H(x_i; t) \frac{f(x_i)}{g(x_i)}$$

$$\text{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i (H(x_i; t) - \hat{\mu})^2$$

$$\text{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i \left(H(x_i; t) \frac{f(x_i)}{g(x_i)} - \hat{\mu} \right)^2$$

Adaptive Importance Sampling



- PDTA requires evaluating POF over a range of times
- With traditional importance sampling, a sampling density must be adapted to the important region(s) for each evaluation time (and each inspection)
- The adaptation process is iterative, and becomes cumbersome when many important regions must be covered

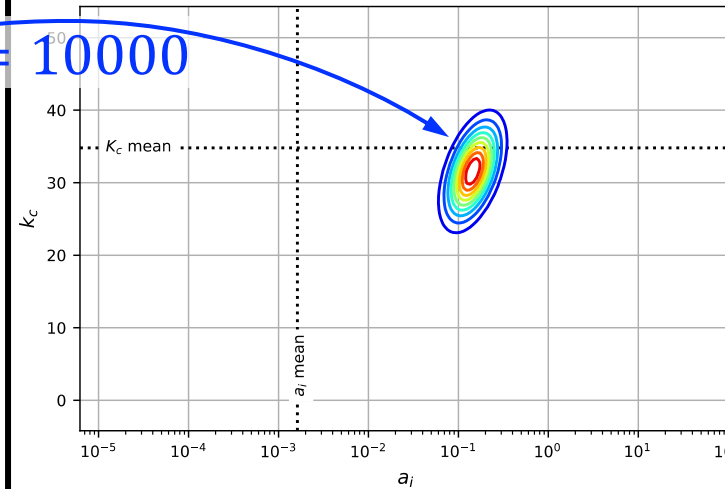
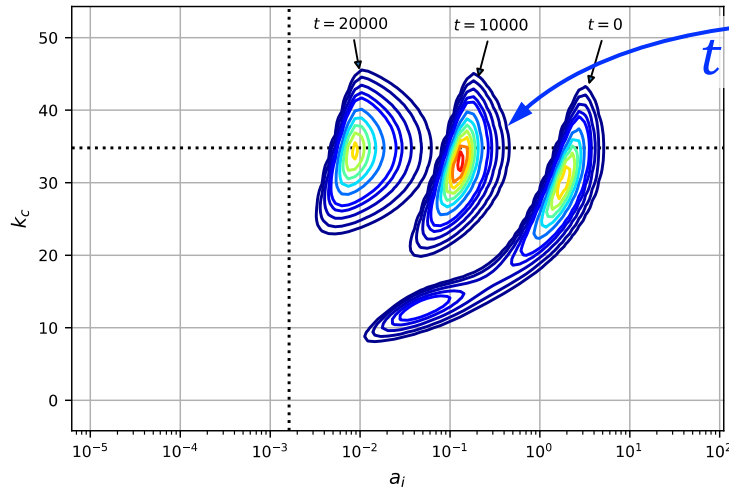
Adaptive Multiple Importance Sampling Approach for PDTA



Important Region(s)

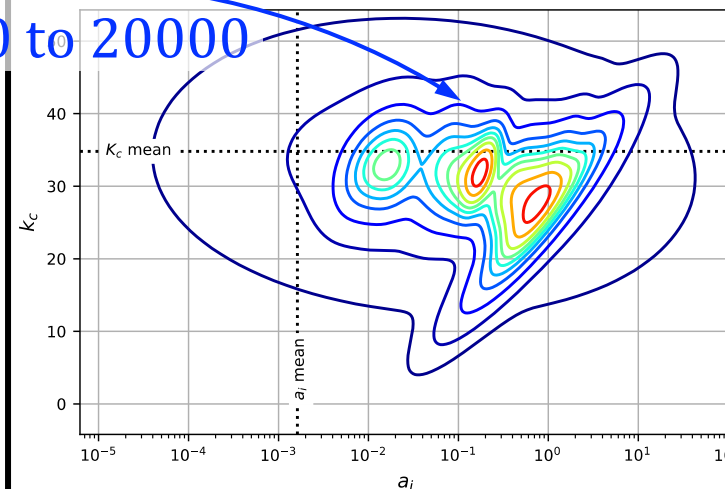
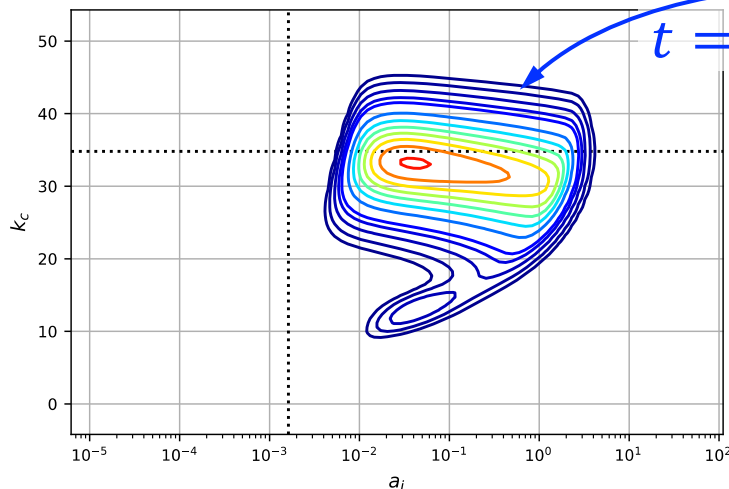
Adapted Sampling Region

Individual Times



- Basic Importance sampling
 - Adapt individual sampling density for each evaluation time

Multiple Times



- Multiple Importance Sampling
 - Adapt a mixture density for the entire range of evaluation times

Going From Collection of Component Densities to Mixture Density



Importance weight for sample x_{ij}

Nominal density

$$w_{mix}(x_{ij}) = \sum_{m=1}^{N_{mix}} \omega_m(x_{ij}) \frac{f(x_{ij})}{q(x_{ij}, \theta_j)} \quad \forall i, j$$

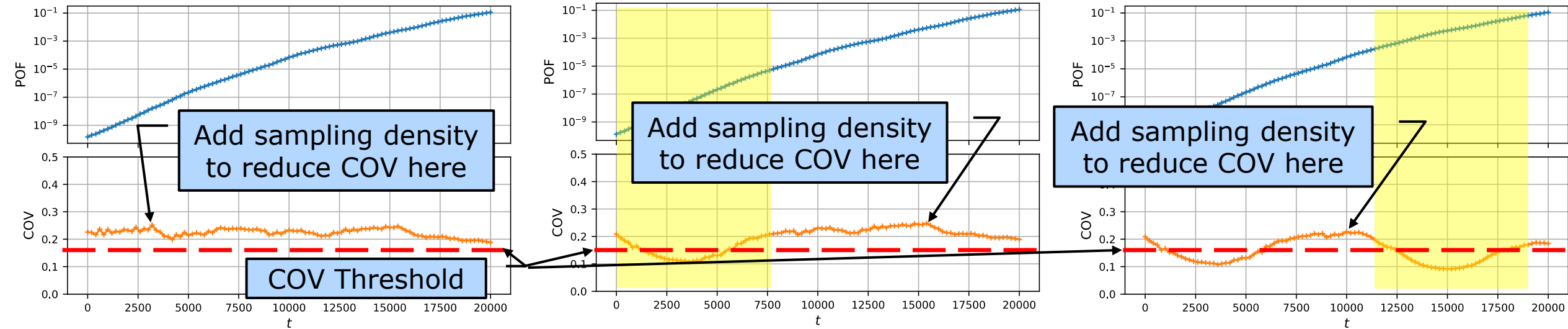
Mixture weights for sample x_{ij} , $\sum_{m=1}^{N_{mix}} \omega_m(x_{ij}) = 1$

j -th component density with parameters θ_j

$$\widehat{POF}(t) = \frac{1}{N} \sum_{i=1}^{N_{samp}} \sum_{j=1}^{N_{mix}} H(x_{ij}; t) w_{mix}(x_{ij})$$

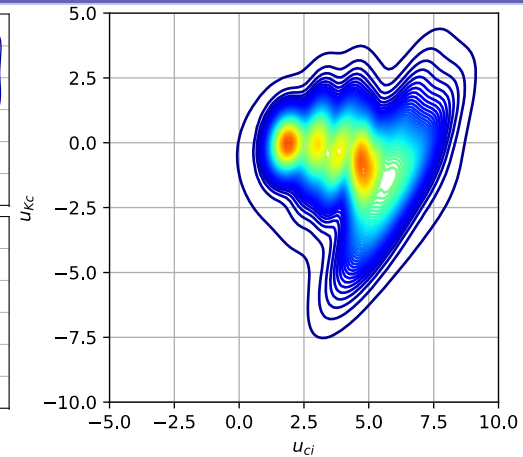
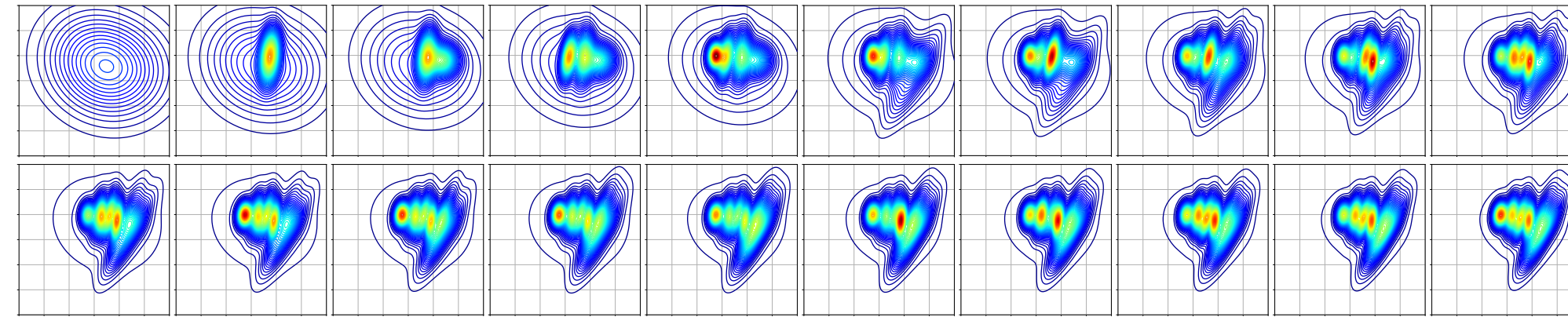
- The collection of individual sampling densities becomes a mixture density by calculating the importance weight of each sample as a weighted sum of the individual sampling density importance weights
- Adds complexity to importance weight calculation (and recalculation), but greatly reduces the number of sample crack growth evaluations

Mixture Density Adaptation

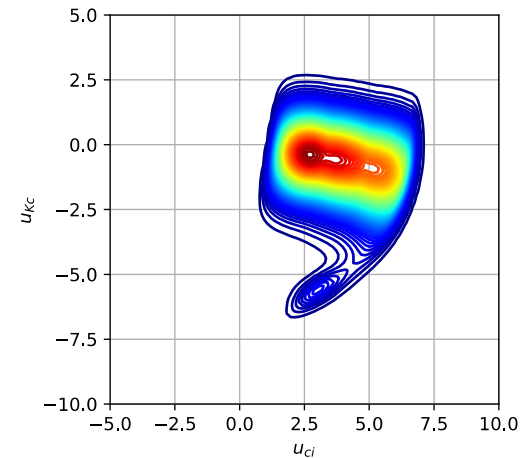


- Coefficient of Variation, $COV = \frac{\widehat{POF}}{\sqrt{\text{Var}(\widehat{POF})}}$, is used to decide if each POF estimate is converged, and to pick which POF estimate (time value) to focus on with the next component density
- Adaptation continues iteratively until all POF estimate COVs are less than or equal to the COV threshold (user specified)

Mixture Density Adaptation



- Above: Progression of Mixture Density adaption - a multivariate normal component density, tailored to the POF(t) estimate with the highest COV, is added at each step
- Top Right: mixture density satisfying the COV threshold for all POF(t) estimates (1500 samples)
- Bottom Right: ideal sampling density (9000 samples – 300x300 grid)



SMART|DT Adaptive Multiple Importance Sampling Setup



SMART|DT

Information

Analysis

Material

Geometry

Loading

Inspections

Run

Results

Analysis

Output Options

Growth

Probabilistic

Method

Adaptive Importance Sampling

Random Seed

2117195382

initializes the pseudo-random number generator for sampling

Target COV

0.2

threshold value at which adaptation will stop adding samples to improve the POF estimate

Samples Per Iteration

100

number of samples generated for each component sampling density

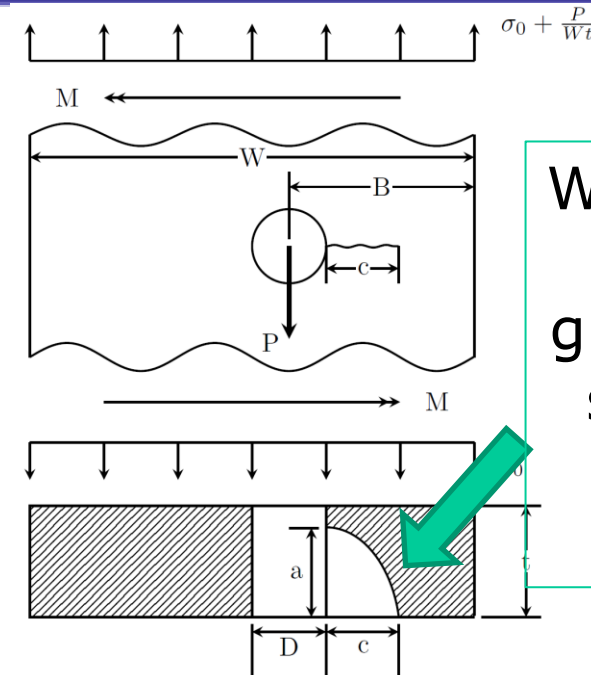
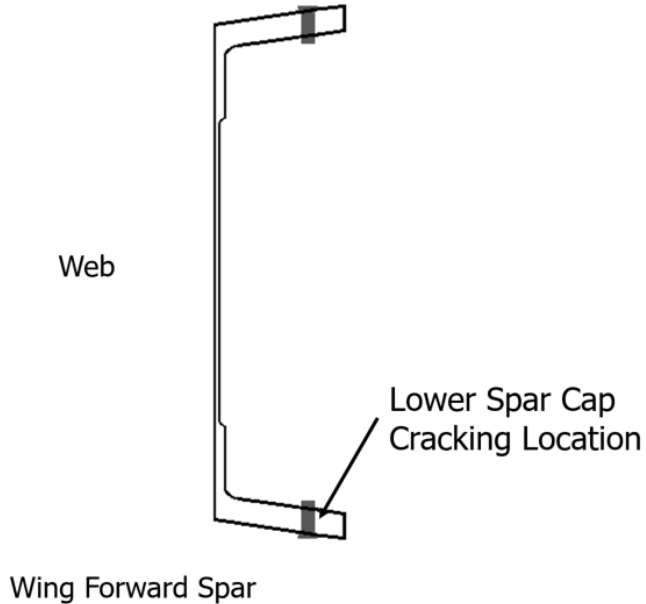
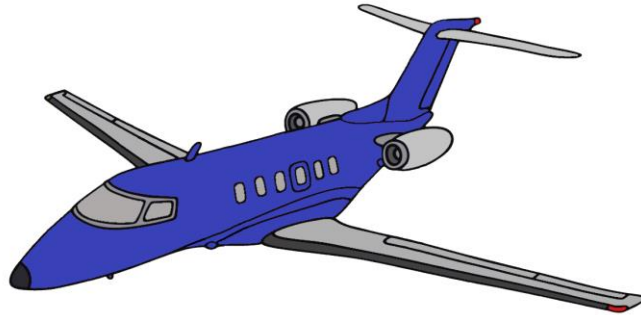
Maximum Iterations

100

number of adaptation iterations at which the adaptation algorithm will stop even if the COV threshold is not met by all POF estimates

- To use Adaptive Multiple Importance Sampling in SMART|DT, go to the *Analysis* page (top navigation), select *Probabilistic* in the side navigation, and change the *Method* to Adaptive Importance Sampling

Adaptive Multiple Importance Sampling Example Problem



We will use HyperGrow to evaluate a crack growth curve for every sample with random da/dn , aspect ratio and hole diameter

Geometric Variables	Value
Hole radius	0.164 in
Cap Thickness	0.175 in
Cap Width	1.80 in



GUI - Information

SMART|DT

Information

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Results

Information

Website

Information

Provide information about the project.

Project Summary

NAME (REQUIRED)

Untitled

DESCRIPTION

Demonstrate Adaptive Multiple Importance Sampling algorithm with HyperGrow and additional random variables

Aircraft Information

MAKE (OPTIONAL)

RR

MODEL (OPTIONAL)

RR45

SERIAL NUMBER (OPTIONAL)

7654

TYPE CERTIFICATE DATA SHEET - TCDS (OPTIONAL)

9887

This program was developed under sponsorship from the Federal Aviation Administration (grants 12-G-012 and 16-G-005) by the University of Texas at San Antonio (UTSA) and partners St. Mary's University, Textron Aviation, Nuss Sustainment Solutions, and Fieldstone Software. The responsible personnel are: Harry Millwater (PI - UTSA), Juan Ocampo (StMU), Beth Gamble (TA), Chris Hurst (TA), Marv Nuss (NSS), JR Lawhorne (Fieldstone), Nathan Crosby (UTSA PhD student), Daniel Ocampo (UTSA MS student), Sohrob Mattighi (Program Manager FAA), Mike Reyer (FAA Kansas City Office).



GUI – Analysis (I)

SMART|DT

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Results

Analysis

Output Options

Growth

Probabilistic

Probability of Failure (POF)

Flights

Maximum Flights Calculation

Flight Units

Analysis

Output Options

Growth

Probabilistic

Model

Source

Crack Model

HyperGROW

GEOMETRY FACTOR

WIDTH

THICKNESS

HOLE DIAMETER DISTRIBUTION

VALUE

WALKER EXPONENT

FAILURE CRITERIA

Hours Per Flight

GUI – Analysis (II)



SMART|DT

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Results

Analysis
Output Options
Growth
Probabilistic

Method
Adaptive Importance Sampling

Random Seed
237206521

Target COV
0.2

Samples Per Iteration
100

Maximum Iterations
100

GUI - Material



SMART|DT

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Category

Custom

Aluminum

Steel

Titanium

Group

2014 Series

2024 Series

2124 Series

2224 Series

7050 Series

7075 Series

7150 Series

7175 Series

7475 Series

Treatment

7475-T7351

7475-T761

7475-T7651

Form, Orientation

Plate L

Plate LT

Plate TL

Summary

Length: Inches

Stress: KSI

Category: Aluminum

Group: 7475 Series

Treatment: 7475-T7351

Form, Orientation: Plate TL

Published data for some of the material properties is unavailable. Material property data is available for Fracture Toughness, Yield Strength and Ultimate Strength. User specified values for Paris Constant and Paris Exponent inputs are needed.

FRACTURE TOUGHNESS

$T = 1.3-4.0$

DISTRIBUTION

Normal

MEAN

37.0

STDEV

3.8

YIELD STRENGTH

DISTRIBUTION

Deterministic

VALUE

57.0

ULTIMATE STRENGTH

DISTRIBUTION

Deterministic

VALUE

70.0

PARIS CONSTANT Log(C)

DISTRIBUTION

Normal

MEAN

-7.888

STDEV

0.02

PARIS EXPONENT

DISTRIBUTION

Normal

MEAN

2.586

STDEV

0.02

PARIS CORRELATION

-0.9

15

GUI – Geometry



SMART|DT

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Results

Equivalent Initial Flaw Size (EIFS)

Category

Custom
Commercial Transport
Military Fighter
Military Transport

Group

Data Set

Summary

Initial Crack Size Distribution

DISTRIBUTION

LogNormal

MEAN

0.0091

STDEV

0.00125

Aspect Ratio

DISTRIBUTION

Normal

MEAN

1.5

STDEV

0.1

The EIFS is traditionally determined through the process of growing in-service or tear-down cracks backwards to time zero. As such, the results are dependent upon the aircraft location, assumed material parameters, and loading history. As a result, it is not recommended to use an EIFS distribution for a different application than for which it was derived. The EIFS values are provided here as a guide and care should be taken to select the distribution that best matches the aircraft mission, joint geometry and manufacturing methods, or ensure that the distribution is appropriately conservative.

GUI - Loading



SMART|DT

 Information

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 **Loading**

 Inspections

 Run

 Results

Extreme Value Distribution (EVD) Method

User Specified EVD

Location
14.5

Scale
0.8

Shape
0

 **Distribution Type: Gumbel**
Maximum Value: Infinite

Note, the EVD is always defined on a per-flight basis.

Constant Amplitude Loading

Maximum Stress
5.5

Cycles Per Flight
55

GUI - Inspections



SMART|DT

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Information

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Analysis

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Material

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Geometry

📈
Loading

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Inspections

▶️
Run

📊
Results

Inspection Presets

Name	Type	Inspection Prob.	Detection Prob.	Repaired Crack
No Presets				

Delete
Edit
Add

Inspections

Flights	Preset	Type	Inspection Prob.	Detection Prob.	Repaired Crack
No Inspections					

Delete
Edit
Add

GUI - Run



SMART|DT

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Run

Results

0% complete.

Start Analysis

✕

DAT File

```

!-----
!   AIRCRAFT INFORMATION
!-----
TITLE = Untitled
AC_MAKE = RR
AC_MODEL = RR45
AC_SERIAL_NUM = 7654
AC_TCDS = 9887
!-----
!   METHOD
!-----
HOURS_PER_FLIGHT = 1.65
INTEGRATION_METHOD = AIS 237206521
AIS_TARGET_COV = 0.2
AIS_NSAMPLES = 100
AIS_MAXITER = 100
POF_MAX_INC = 30000 500
!-----

```

Analysis Details

```

40 % complete.
50 % complete.
60 % complete.
70 % complete.
80 % complete.
90 % complete.
100 % complete.

*****
***** PDTA analysis complete *****
*****

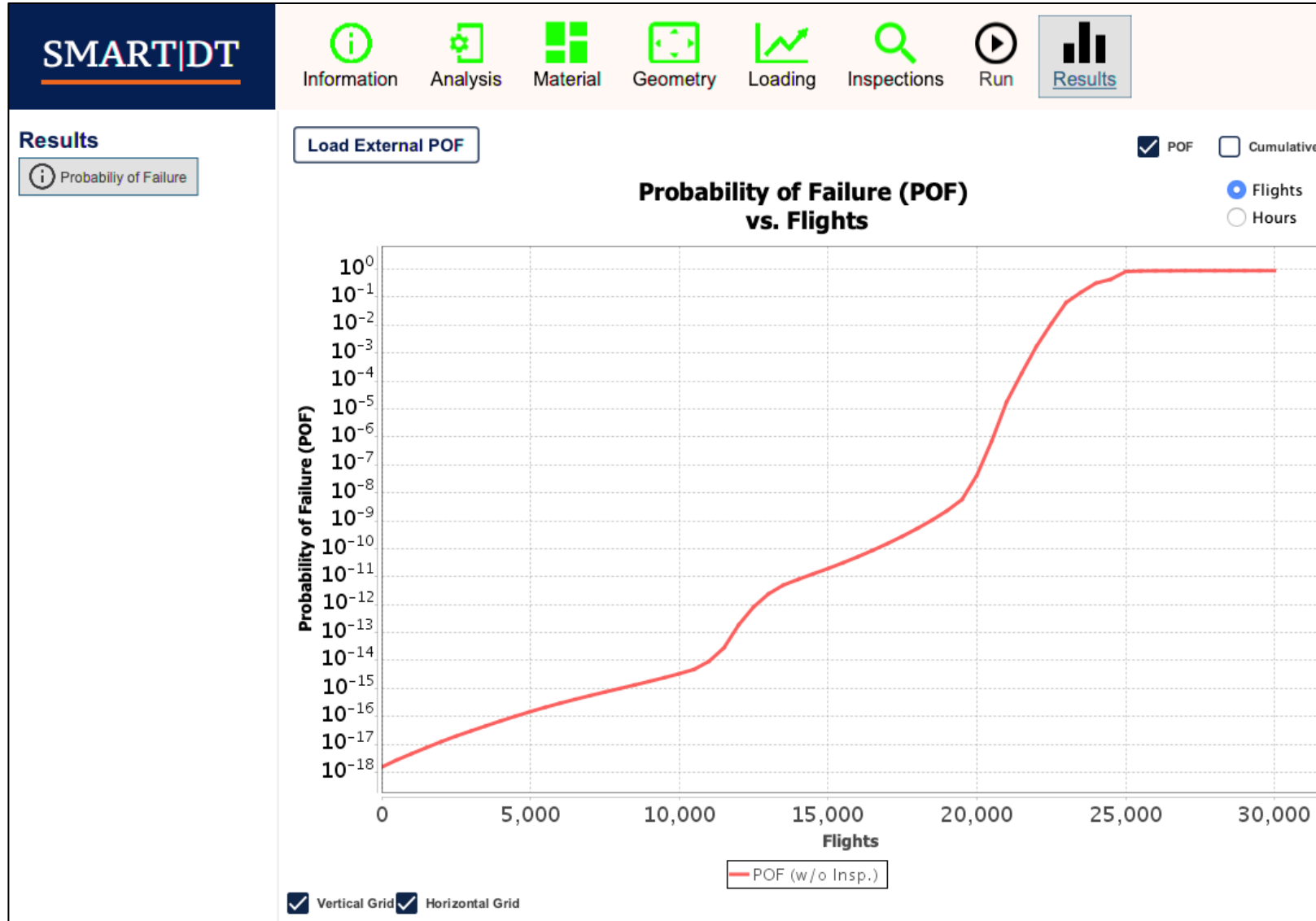
Total CPU time =      1.349 secs
Total wall time =      0.311 secs

```

Show/Export

Finished in 0.3 seconds

GUI - Results



Summary



- Introduce Adaptive Multiple Importance Sampling now available in SMART|DT
- This method reduces PDTA runtimes by 6 orders of magnitude compared to Standard Monte Carlo with 10^9 samples
- The speed of the method allows da/dn variation to be included in the analysis

Questions

